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**Кассем Аттиа Ибрагем Кассем**

# Волновые процессы, магнитные поля и нелинейные структуры в пылевой плазме в окрестностях Луны

**Специальность:** 1.3.9 - Физика плазмы

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# Wave Processes, Magnetic Fields and Nonlinear Structures in Dusty Plasma in the Vicinity of the Moon

**Specialty:** 1.3.9 - Plasma Physics

**SYNOPSIS**
of the dissertation for Academic Degree of Doctor of Philosophy in Physics and Mathematics

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**Leading organization::**

The defense will take place on **00 00 2023 at 00:00 am** at the meeting of the dissertation council, created in the Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)" (MIPT, Phystech)

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The text of the dissertation is available in the library MIPT, Phystech or on the website https://mipt.ru

Abstract was sent « » ____________ 2023 г.

**Scientific secretary of the Dissertation Council** **Kuzmichev Pavel Konstantinovich**

# Description of the dissertation work

**Relevance of the research topic.** Dust particles are considered to be the main significant components in cosmic plasma system. Perhaps there is only one exception to this rule, the Sun itself and the region in its immediate vicinity, where, due to high temperatures, dust cannot exist. Nano- and microscale dust particles are found in various locations throughout the solar system. These particles are present in interplanetary space, as well as in the plasma of the ionospheres and magnetospheres of planets. Additionally, they can be found in planetary rings and in the vicinity of cosmic bodies that lack their own atmosphere, such as Mercury, asteroids, and comets, etc. Furthermore, the study of dust particles in space dusty plasma systems is a rapidly growing field of research [1–4]. In reality, the charge on the dust grain (DG) is considered an important feature that occurs as a result of a number of processes, among them the electron and ion currents flowing into or out of the dust grain, as well as other processes like secondary emission, photo-emission of electrons, etc. These processes lead to dust charge fluctuations. The presence of massive (compared to ions and electrons) charged dust grain varies the characteristic of both spatial and temporal scales in plasma, and in some situations even gives rise to a new physics of certain phenomena. Furthermore, the presence of such fairly massive (depending on the current plasma parameters) charged dust grains modifies the spectra of waves propagating in the medium, determines the nature of the development of instabilities and nonlinear processes, and so on.

In recent years, there has been a significant increase in interest in the direct study of cosmic dust particles. The NASA's Stardust mission was undertaken to collect and deliver to Earth particles from the vicinity of the nucleus of comet 81P / Wild 2. The project was successfully implemented on January 15, 2006, when a capsule with the samples of cometary matter returned to Earth [5]. In the recent American mission LADEE [6], lunar dust was studied using observations from orbit. The Russian Luna-25 and Luna-27 missions are being prepared to investigate the properties of dusty plasma above the LS [7]. That is why, in our dissertation, while discussing the properties of dust/dusty plasma in the space around the Sun, we will focus on its state above the lunar surface (LS). Several theoretical studies on the dust particles in dusty plasma (DP) transport and dynamics above the Moon have been performed by many authors not taking into account magnetic fields (for instance, [8, 9]). The model [7, 10] of the electrostatically produced dusty plasmas does not take into account also nonstationary processes associated with the finiteness (in time) of daylight hours on the Moon. Another kind of nonstationary processes is related to the dust grain charge variations [11, 12], which can result in anomalous dissipation playing often the decisive role in dusty plasmas. The importance of the processes related to the dust grain charge variations in the electrostatically produced dusty plasmas at the Moon has not yet been clarified. Besides, linear and nonlinear waves research into dusty plasma system above the Moon was preformed in the absence of the magnetic field [13, 14]. Kopnin and Popel [15] demonstrated the possibility of the existence and propagation of dust acoustic solitons in the dusty plasma of the lunar exosphere. In reality, magnetic fields can exist. Approximately one-fourth of the lunar orbit passes through the Earth's magnetotail, which contains very rarefied plasmas in the geomagnetic tail wings as well as hotter and denser plasmas in the plasma sheath. However, studies examining this trend are scarce. In addition, there are the so-called areas of magnetic anomalies on the Moon associated with magnetic matter in the lunar crust. The measurements of near-surface magnetic fields on the visible side of the Moon, performed as part of the Apollo 12, 14, 15, and 16 missions, amounted to $3.8 \times 10^{-4}$, $1.03 \times 10^{-3}$, $3 \times 10^{-5}$, and $3.27 \times 10^{-3}$ G, respectively [16]. As a result, the main motivation in this work is to investigate the dynamics of dust grains under the influence of the magnetic field of the Earth's magnetotail above the sunlit side of the Moon (SSM), as well as to clarify the importance of processes related to dust grain charge variations in electrostatically produced dusty plasmas. In addition to studying the properties and features of linear and nonlinear wave processes and their stability above SSM. This study provides a developing macroscopic picture of the dust grain dynamics and dusty plasma above the Moon, which may be useful for future lunar missions, particularly Roscosmos missions (like Luna-25, Luna-26, and Luna-27) which will include experimental research on lunar dust and dusty plasma both near the LS (Luna-25 and Luna-27) and in orbit (Luna-26) around the Moon.

## The objectives of the dissertation

The purpose of the dissertation work is to build a developed picture of the influence of magnetic fields on dusty plasma in the vicinity of the Moon, including wave processes and nonlinear structures. To achieve this aim, we find it necessary to implement more specific tasks:

- Describe and model the effect of the Earth's magnetotail magnetic fields and lunar magnetic anomalies on the dynamics of charged dust particles above the Moon.
- Investigate the trajectories of dust grains above the lunar surface in the case of the processes of variation in the charges of dust grains and their oscillations during the lunar day.
- Investigate wave processes of the lower-hybrid turbulence and the development of the lower-hybrid turbulence excitation which is due to the interaction of the Earth's magnetotail with the lunar dusty plasma.
- Derive and analyze nonlinear partial differential equations such as the modified Kadomtsev-Petviashvili equation and the modified Zakharov-Kuznetsov equation, which characterize the nonlinear dynamics of wave structures in dusty

plasma above the Moon's illuminated side. The modified Kadomtsev-Petviashvili equation describes the nonlinear dynamics of wave structures in dusty plasma above the Moon's lighted side in the situation in which localization along the magnetic-field vector is much stronger than along other directions. Furthermore, the modified Zakharov-Kuznetsov equation describes the dynamics of nonlinear wave structures in the dusty plasma above the Moon's lighted side in the case of low frequencies and the pancake-like shape of the wave packet in the direction along the external magnetic field.

## Scientific Novelty

- For the first time, the influence of magnetic fields of the Earth's magnetotail and of the magnetic fields in the regions of magnetic anomalies of the Moon on the transport of dust above the lunar surface and on the formation of dusty plasma above the Moon is performed. A new qualitative effect that does not exist in the absence of the magnetic field is provided. The magnetic component of the Lorenz force transports the dust grains from the region of high lunar latitudes toward the equator of the Moon, which is due to the magnetic fields of the Earth's magnetotail.

- For the first time, the investigation of the dust grain trajectories above the lunar surface shows that the processes of variation in the charges of dust grains, leading to the attenuation of their oscillations over the lunar surface, are too fast in comparison with the day duration on the Moon. In this connection, most of the dust in the dusty plasmas above the Moon's illuminated side can be considered as levitated.

- For the first time, analysis of linear and nonlinear processes is performed for the lower hybrid waves, which can be important in the dusty plasma over the Moon. It is shown that the lower-hybrid turbulence can be generated wherever the Earth's magnetotail interacts with the near-surface dusty plasmas above the Moon. The electric fields generated by the development of lower-hybrid turbulence are evaluated.

- For the first time, in application to the lunar dusty plasma, a nonlinear modified Kadomtsev-Petviashvili equation is obtained to describe two-dimensional nonlinear dust acoustic structures above the Moon. Stability analysis of the one-dimensional soliton solution of modified Kadomtsev-Petviashvili equation showed that it is stable.

- For the first time, the modified Zakharov-Kuznetsov equation is derived to describe the dynamics of nonlinear structures in the dusty plasmas above the Moon in the case of low frequencies and pancake-like shape of the wave packet in the direction along the external magnetic field above the Moon's illuminated side.

## Scientific Statements for the defense

- The physical and mathematical model is developed to describe the dynamics of charged dust particles above the Moon under the effect of the magnetic fields of the Earth's magnetotail and the lunar magnetic anomalies. It is shown that due to the action of the magnetic fields, transfer of charged dust particles is possible over long distances above the Moon's surface, and the dusty plasma above the day-side of the Moon can exist in the entire range of lunar latitudes.

- It is shown that most of the dust particles in dusty plasma above the Moon's illuminated side are levitated. Only very fine particles (with sizes smaller than several dozens of $\mu$m) can be treated as lofted. This is, in particular, related to a fast process of charge variation of dust particles in comparison with the day duration on the Moon.

- It is shown that when the Earth's magnetotail interacts with dusty plasmas near the lunar surface, the excitation of lower-hybrid turbulence is possible. The effective collision frequency, which determines the loss of ion momentum as a consequence of the interaction between magnetotail ions and lower-hybrid waves is determined. The typical electric fields are estimated, which appear due to lower-hybrid turbulence development. Such electric fields can make a significant contribution to the total electric field above the lunar surface, which should be considered in in further experimental research on electric fields above the Moon.

- A two-dimensional description of nonlinear waves in dusty plasmas above the Moon is given. In this connection, the modified Kadomtsev-Petviashvili and Zakharov-Kuznetsov equations are derived under the conditions of the lunar dusty plasmas and analyzed. Soliton solutions of these equations are obtained. It is shown that the one-dimensional soliton solutions of these equations are stable.

## Scientific and practical significance of the work

The dissertation results can be used by a wide range of specialists involved in the study of physical characteristics of dusty plasma, as well as contribute to further theoretical and practical research on dusty plasma physics above the Moon or in space.

The dynamics of the dust grains above the lunar surface and the processes of charge variations of dust grains under the influence of the magnetic fields of the

Earth's magnetotail and magnetic anomalies may be helpful in future lunar missions. For instance, the future Roscosmos missions, Luna-25, Luna-26, and Luna-27, will include experimental research on lunar dust and dusty plasma both near the lunar surface (Luna-25 and Luna-27) and at the orbit (Luna-26) around the Moon.

The solitary wave study above the Moon can be contributed to interpretation from the point of view of the description of the so-called transient lunar phenomena representing short-lived light, changes in color or appearance on the surface of the Moon. Phenomena of this kind are sometimes related to the sporadic release of gases on the Moon. However, it seems feasible to suggest that upon propagation of electrostatic solitons along the lunar surface, glow can be caused by plasma emission from the region of soliton localization, i.e., solitons can contribute to the explanation of these or other transient lunar phenomena.

## Author's Contribution

The author participated in the statement of the problems, performed analytical and numerical calculations and mainly participated in formulation of the statement of conclusions.

## Approbation of the work

The main results of the dissertation were presented and discussed at seminar at the Space Research Institute (IKI, RAS), as well as 9 oral and poster presentations at international conferences and Russian conferences.

- XXXV International Conference on Equations of State for Matter, 1- 6 March 2020, Elbrus, Russia.
- 64th International MIPT Scientific Conference, 29 November – 3 December 2021, Dolgoprudny, Moscow region, Russia.
- 18th and 19th International workshop complex systems of charged particles and their interactions with electromagnetic radiation, 2022 and 2023, GPI RAS, Moscow, Russia.
- XVII Conference of Young Scientists "Fundamental and applied space research", 30 September - 2 October 2020, IKI RAS, Moscow, Russia.
- The Twelfth Moscow Solar System Symposium, 11-15 October 2021, IKI RAS, Moscow, Russia.
- The first international conference on space education: The Road to Space, 5-8 October 2021, IKI RAS, Moscow, Russia.

- 9th International Conference on the Physics of Dusty Plasmas ICPDP, 23– 27 May 2022, IKI RAS, Moscow, Russia.

## Publications on the thesis topics

Based on the dissertation materials, six publications have been published and indexed in Scopus and Web of Science databases.

## The structure of the thesis

The dissertation consists of an introduction, four chapters, and a conclusion summarizes results and achievements of the dissertation work. In the introduction, the work's goals, scientific novelty, practical significance, research techniques, and provisions submitted for defense are provided along with the relevance of the work.

**The $1^{st}$ chapter** of the dissertation work provides some introductory facts about dust particles and is followed by a scientific literature review on the observation and formation of dust particles in space. This chapter discusses the sources and creation of dust particles as well as their charging above the Moon. The significance of studying the influence of the magnetic fields of the Earth's magnetotail and lunar magnetic anomalies on the dynamics of the DGs above the Moon and waves in dusty plasma is presented in this chapter.

**In Chapter 2**, the possible effect of the magnetic field of the Earth's magnetotail and the magnetic field in the regions of magnetic anomalies on the formation of dusty plasma above the Moon is determined. For this purpose a physical-mathematical model for a self-consistent description of dust grains and photoelectrons in the presence of the magnetic field is developed. The near-surface dusty plasma of the Moon is quite rarified, therefore, the influence of neighboring dust grains on each other can be ignored. Therefore, the dynamics of a dust grain above Moon's surface is determined by the Newton's second law, taking into account the electrostatic and magnetic components of Lorenz force and the gravity force:

$$m_d \frac{d^2\mathbf{r}_d}{dt^2} = q_d\mathbf{E} + \frac{q_d}{c}\mathbf{v}_d \times \mathbf{B} + m_d\mathbf{g}_0, \tag{1}$$

$$\frac{dq_d}{dt} = I_e(q_d) + I_i(q_d) - I_{ph}(q_d) + I_{e,ph}(q_d), \tag{2}$$

here, $m_d$ is the dust grain mass, $\mathbf{r}_d$ is the dust grain radius-vector, $\mathbf{E}$ is the electric field, $c$ is the velocity of light, $\mathbf{v}_d$ is the dust grain velocity, $\mathbf{g}_0$ is the gravity near the lunar surface, $I_e(q_d)$ and ionic $I_i(q_d)$ are microscopic currents of the solar wind electrons and ions, respectively, as well as $I_{ph}(q_d)$ is the photoelectrons current, and the reverse current of photoelectrons $I_{e,ph}(q_d)$ knocked out from neighboring dust particles contributes also to the total current. Moreover, the mathematical expression of these currents are given as the following:

$$I_e \approx -\pi a^2 e n_{eS} \sqrt{\frac{8T_{eS}}{\pi m_e}} \left(1 + \frac{Z_d e^2}{a T_{eS}}\right), \tag{3}$$

$$I_{ph} \approx -\pi a^2 e N_0 \sqrt{\frac{T_{e,ph}}{2\pi m_e}} \times \left(1 + \frac{Z_d e^2}{a T_{e,ph}}\right) \exp\left\{\left(-\frac{Z_d e^2}{a T_{e,ph}}\right)\right\}, \tag{4}$$

$$I_i \approx \pi a^2 e n_{iS} \sqrt{\frac{T_{iS}}{2\pi m_i}} \frac{u_{Ti}}{u_i} \times \left\{ \frac{u_i + u_0}{u_{Ti}} \exp\left\{\left(-\frac{(u_i - u_0)^2}{2u_{Ti}^2}\right)\right\} + \frac{u_i - u_0}{u_{Ti}} \exp\left\{\left(-\frac{(u_i + u_0)^2}{2u_{Ti}^2}\right)\right\}\right\} + \pi a^2 e n_{iS} \sqrt{\frac{T_{iS}}{4m_i}} \frac{u_{Ti}}{u_i} \left\{ \mathrm{erf}\left(\frac{u_i + u_0}{\sqrt{2} u_{Ti}}\right) + \mathrm{erf}\left(\frac{u_i - u_0}{\sqrt{2} u_{Ti}}\right)\right\} \left(1 + \frac{2Z_d e^2}{a T_{iS}} + \frac{u_i^2}{u_{Ti}^2}\right), \tag{5}$$

$$I_{e,ph} \approx -\pi a^2 e n_{e,ph} \sqrt{\frac{8T_{e,ph}}{\pi m_e}} \left(1 + \frac{Z_d e^2}{a T_{e,ph}}\right), \tag{6}$$

here, a is the size of DG, $Z_d$ is the dust grain charge number ($q_d = Z_d e$), $n_{e(i)S}$ is the solar wind electron (ion) number density, $T_{e(i)S}$ is the temperature of the solar wind electrons (ions), $m_i$ is the ion mass, $u_0 = \sqrt{2Z_d e^2/am_i}$, $u_{Ti} = \sqrt{T_{iS}/m_i}$ is the thermal velocity of the solar wind ions, $u_i$ is the solar wind velocity, $N_0$ is the concentration of photoelectrons near Moon's surface at its equator and $n_{e,ph}$ is the photoelectron number density. Expressions (3)-(6) are derived for the case of positive dust grain charges. We assume that the dust grain surfaces and the lunar surface have the same work functions. In this connection, it is possible to present Eq. (4) (for $I_{ph}$) in the form when it does not contain a factor expressed in terms of the characteristics of the solar radiation spectrum. In the situation considered, this factor can be written with the use of the quantity $N_0$. Eq. (5) was derived specially for the case of arbitrary ion flow speeds [17].

When solving Eqs.(3)-(6), it is necessary to take into account the following expression for the vertical component of the electric field $E$ formed by the charged surface of the Moon depending on height $h$ above the LS. The electric field strength is ([18])

$$E(h,\theta) = \frac{2T_{e,ph}}{e} \frac{\sqrt{\cos\theta/2}}{\lambda_D + h\sqrt{\cos\theta/2}}, \quad (7)$$

where $\lambda_D$ is the Debye length of photoelectrons near Moon's surface and $\theta$ is the subsolar angle. Note that angle $\theta$ for smooth lunar surface (without hills and depressions) is approximately equal to the lunar latitude because the angle between Moon's axis and ecliptic plane is merely 1.5424°. The dependence of the electric field on the subsolar angle in Eq. (7) is related to the dependence of the amount of photons absorbed by the lunar surface on the angle $\theta$. The dusty plasma parameters above the sunlit surface of the Moon depend on the subsolar angle $\theta$ which (for a flat lunar surface without any hills and depressions) is approximately equal to the lunar latitude. This is caused by the fact that the angle formed by the axis of the Moon and the ecliptic plane is only $1.5424^o$. There is a definite critical value of the subsolar angle depending on the dust grain size $a$ and exceeding $\theta = 75.52^o$. Only in a very limited range of subsolar angle $\theta$ ($|\theta| > \theta_0$) the dust grain ascent is possible, where $\theta_0$ is determined from the relationship [19]

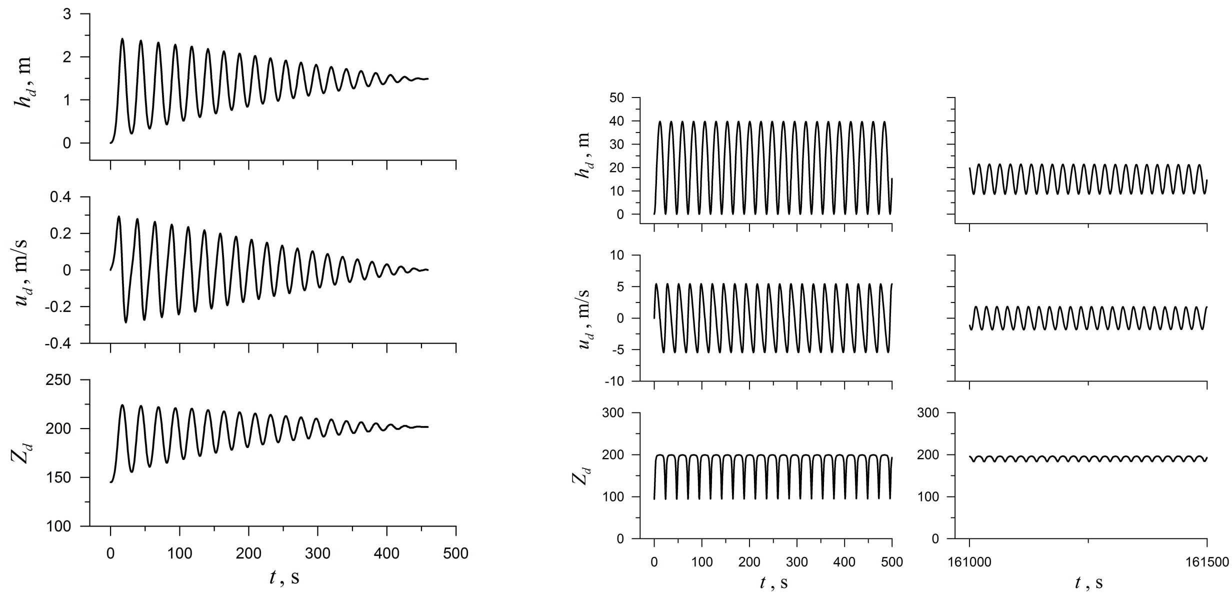
$$\sqrt{\cos\theta_0}|\ln(4\cos\theta_0)| = \frac{8\sqrt{2}\pi^2 a^2 \rho^2 G R_M \lambda_D}{9} \left(\frac{e}{T_{e,ph}}\right)^2, \quad (8)$$

where $\rho$ is the lunar regolith density, $G$ is the universal gravitational constant, $R_M$ is the radius of the Moon, $\lambda_D$ is the Debye length of photoelectrons near the lunar surface, $T_{e,ph}$ is the photoelectron temperature, $e$ is the elementary charge. If we restrict ourselves to considering the dust grains with sizes not exceeding 1 $\mu$m (i.e., the grains typical for the near-surface layer at the sunlit surface of the Moon [7, 10, 20]), then we find that the subsolar angle critical value is not larger than $\theta = 76.14^o$. In the first approximation, we neglect the effects of the magnetic field (instead of Eq. (1)) and solve the following equation

$$m_d \frac{d^2h}{dt^2} = q_d E(h,\theta) - m_d g_0, \quad (9)$$

we use these set of equations to determine the trajectories of motion of dust grains and the dependencies characterizing their motion. Figure 1a presents the lifting altitude $h_d$, the velocity $u_d$, and the charge number $Z_d$ of a dust grain of the radius $a = 0.105$ $\mu$m for the subsolar angle $\theta = 87^o$. The dust grain motion is oscillatory around a stable position $h_{d0} = 148$ cm, $u_{d0} = 0$, $Z_{d0} = 202$. The oscillation period is 26.8 s. The oscillations are damped.

Figure 1b presents the dependencies characterizing the trajectory of motion of a smaller dust grain with the radius $a = 0.06823$ $\mu$m at $\theta = 87^o$. We determine $h_{d0} = 1437$ cm, $u_{d0} = 0$, $Z_{d0} = 192$. Again the dust grain motion above the




**(a)** $a = 0.105$ $\mu m$ **(b)** $a = 0.06823$ $\mu m$

**Figure 1:** *Dependencies characterizing the trajectory of motion of a dust grain with the varying charge above the lunar surface at* $\theta = 87^o$*.*

lunar surface is oscillatory (with an oscillation period of 23.6 s), but the damping of the oscillations is weaker than in the case of a larger dust grain. The weakly damped character of the oscillations in this case is related to the fact that the stable equilibrium position is attained at a sufficiently high altitude where the effect of photoelectrons on the charge of the dust grain is small in comparison with those of the solar wind and solar radiation. Using the log-normal size-distribution for the lunar regolith grains, which is found on the basis of the observational data from [21] to dust grain sizes of the order of 0.01 $\mu$m, we determine that the damping time of oscillations for most ($\sim$ 83%) of the dust grains (with radii larger than 0.068 $\mu$m and less than 0.105 $\mu$m) which are able to rise over the lunar surface is shorter than the daytime. This means that most of the dust grains above the lunar surface can be considered as "levitated" those. For the dust grains with the sizes smaller than 0.068 $\mu$m the oscillation character of their motion is manifested during the entire day on the Moon, and such small grains can be treated as "lofted" those.

The scheme of the Moon's motion in Earth's magnetotail, the magnetic field induction vector $B$, the velocity of dust grain $v_d$, and the magnetic component of Lorenz force $F_L$ are shown in Fig. 2. To determine the influence of the magnetic field in the magnetotail on the dust grains above the LS, we are using the equations (1-6) that describe the dynamics and charge of the dust grain in the near-surface layer of the Moon. In the direction perpendicular to the lunar surface, the forces acting on the dust grain are the vertical projection of the magnetic part of the Lorentz force ($\mathbf{F}_L$), the electrostatic force ($q_d\mathbf{E}$), and the gravitational force ($m_d\mathbf{g}_0$). Because the magnetic part of the Lorentz force is much smaller than the electrostatic force or the gravitational force, the vertical position of the dust grain is determined within the set of equations (2) and (9) (using the first approximation where the magnetic

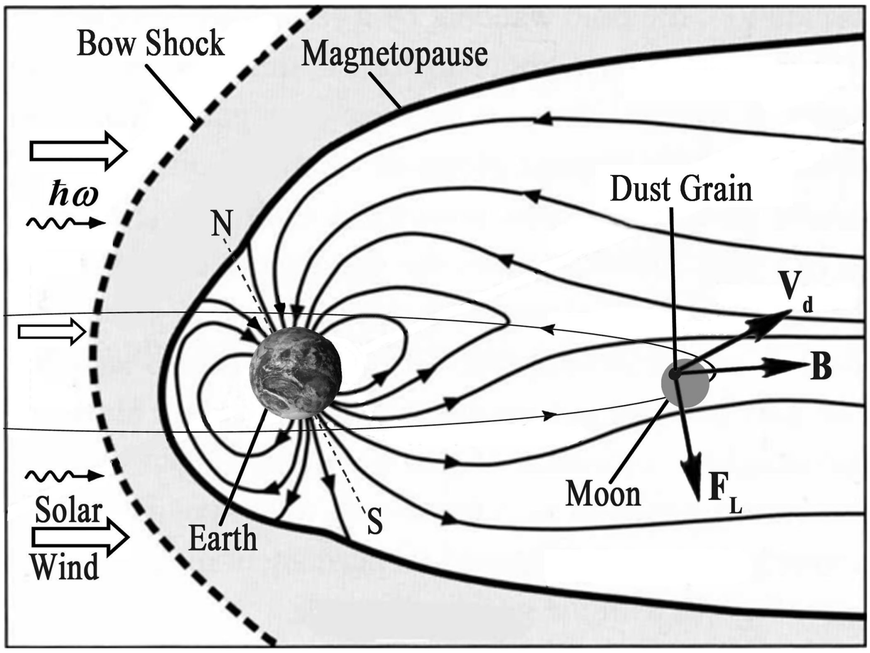



**Figure 2:** *The Moon in the Earth's magnetosphere (scheme). Lunar orbit is depicted by a thin line; arrows on the lunar orbit show the direction of the motion of the Moon. The solar wind and photons of solar radiation ($\hbar\omega$) are also depicted. Directions of the magnetic part of the Lorentz force ($\mathbf{F}_L$), the magnetic field ($\mathbf{B}$), and the dust grain speed ($\mathbf{v}_d$) are given as vectors applied to the dust grain.*

field is neglected as described in the previous section).

In the horizontal direction, the DG is subjected to the action of the uncompensated horizontal projection of the magnetic part of the Lorentz force that results in the dust grain transport towards South. The equation of motion of the dust grain along the lunar surface is

$$R_M \frac{d^2\theta}{dt^2} = -\frac{Z_d e v_d B}{c} \cos\theta, \tag{10}$$

where $R_M$ is the lunar radius, $v_d \approx 1$ km/s is the dust grain speed (approximately equal to the velocity of the Moon along its orbit). In Eq. (10) $\theta$ denotes the latitude (which is equal approximately to the subsolar angle under the conditions of the Moon). The results of numerical calculations within the set of equations (2), (9), and (10) are presented in Fig. 3. The results of numerical calculations are presented in Figs. 3. In calculations, we assume that there is a balance between electrostatic and gravitational forces in the equation (1), i.e. the particle levitates at a certain height, and we also neglect the inclination of the axis and the inclination of the orbit to the ecliptic plane for both the Earth and the Moon. Then, defining $t = 0$ as the moment of time corresponding to the entry of the Moon into the tail of the Earth's magnetosphere. Figure 3 shows the values that characterize the motion of dust grains of different sizes that levitate above the LS under the action of the Earth's magnetic field for various moments of separation of dust particles from the lunar surface (and, accordingly, the beginning of levitation) are shown, $n_{eS} = n_{iS} = 8.7$

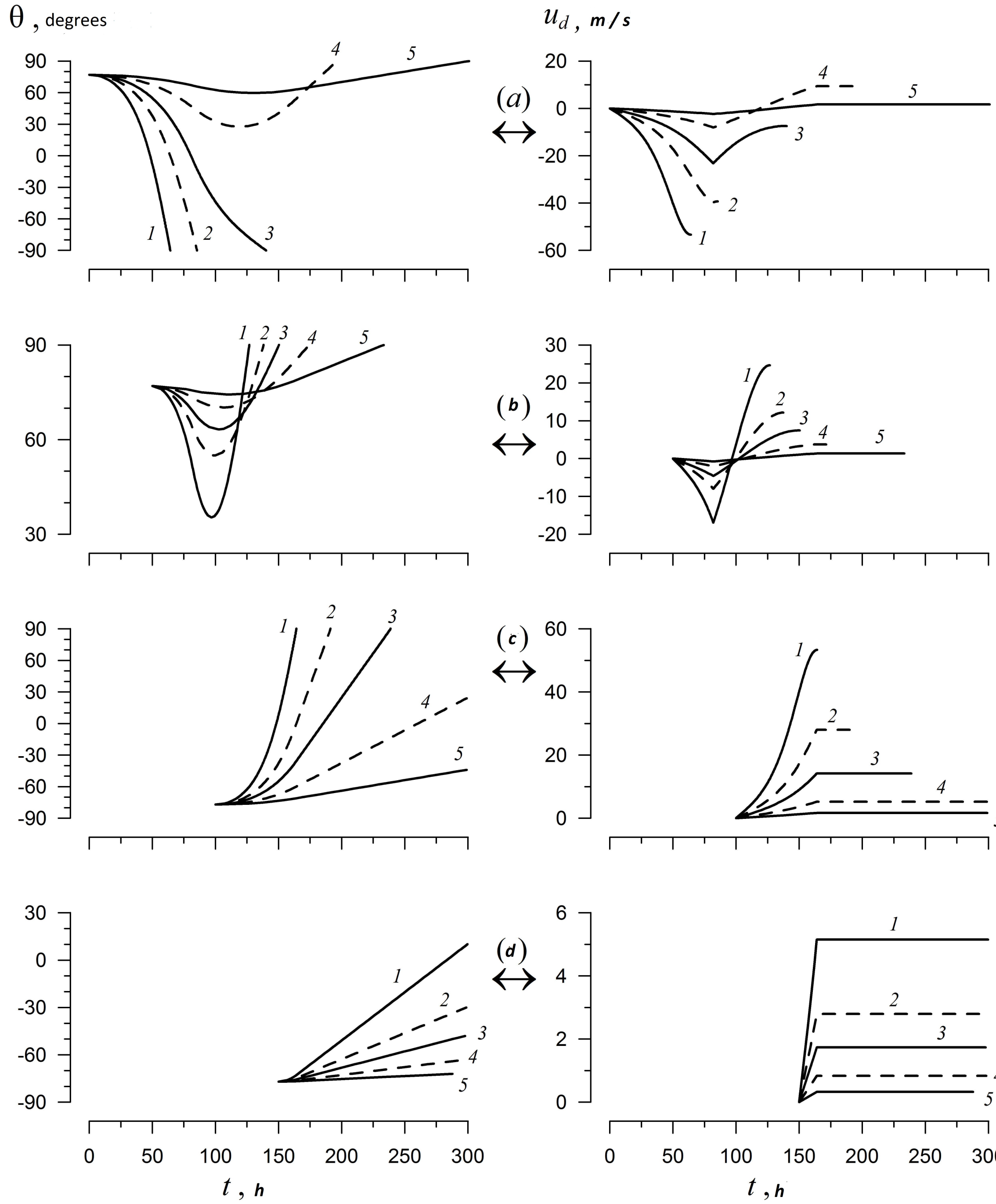



**Figure 3:** *Dependence on time $t$ of the angular coordinate $\theta$ and the velocity component $u_d$ of a dust particle along the lunar surface. Time $t = 0$ corresponds to the entry of the Moon into the tail of the Earth's magnetosphere. Curves (1), (2), (3), (4), and (5) characterize dust particles whose radii are 0.03, 0.04, 0.05, 0.07, and 0.1 $\mu$m, respectively. The results are given for different moments of separation of dust grains from the Moon's surface: 0 (a), 50 (b), 100 (c), and 150 (d) hours.*

cm$^{-3}$, $T_{eS} = 12$ eV, $T_{iS} = 6$ eV, $u_i = 468$ km/s, $|\mathbf{B}| = 10^{-4}$ G, $W = 6$ eV, $T_{e,ph} = 1.9$ eV, $N_0 = 2.9 \times 10^2$ cm$^{-3}$. These values $T_{e,ph}$ and $N_0$ (see [20]) correspond to the solar maximum and the lunar regolith quantum yield given in [22]. From Fig. 3 shows that due to the action of magnetic fields in the tail of the Earth's magnetosphere, charged dust particles can be transported over the lunar surface over long distances and, accordingly, dusty plasma above the Moon's surface illuminated by the Sun can exist for the entire range of lunar latitudes (from $-90^o$ to $90^o$). The transfer of dust particles from the region of lunar latitudes adjacent to the lunar poles ($|\theta| > 76^o$) to the Moon's equator due to the uncompensated magnetic part of the Lorentz force is a new qualitative effect that does not exist in the absence of a magnetic field. Note that the transfer of dust particles is accompanied by changes in their charges.

To analyze the influence of magnetic fields in the regions of magnetic anomalies on the evolution of the dusty plasma system, it is important that both the magnetic field and the dusty plasma in the situation under consideration are "attached" to the lunar surface. The velocity included in the magnetic part of the Lorentz force in this case is of the order of $u_d$, shown in Fig. 3, i.e. $\sim 10$ m/s, in contrast to the situation of the magnetic field of the Earth's magnetosphere, where the dust particle velocity relative to the magnetic field is $\sim 1$ km/s. Thus, the magnetic part of the Lorentz force acting on a dust particle for the fields of magnetic anomalies is either less than or comparable to the analogous force calculated for the magnetic fields of the Earth's magnetotail in the Moon's orbit. In this case, magnetic fields in the regions of magnetic anomalies can lead to changes in the trajectories of dust particles, deviating them from those shown in Fig. 3. Since the characteristic dimensions of the regions of magnetic anomalies are only a few tens of kilometers (see, for example, [23]), the general trend of the motion of dust particles shown in Fig. 3 is maintained.

**The $3^{rd}$ chapter** is dedicated to study a possibility of excitation of lower-hybrid turbulence in dusty plasmas at the Moon. The main components of the dusty plasma near the Moon are charged dust grains, magnetospheric electrons and ions, electrons and ions of the solar wind and photoelectrons formed as a result of the photoelectric effect caused by dust grains levitating and the lunar surface [24]. The velocity of the Moon and near-Moon dusty plasma relative to the magnetic tail plasma are about 1 km/s. Furthermore, during geomagnetic storms and substorms, particles with energies of about 10 keV trapped in the radiation belts can penetrate the magnetotail [21], thus creating charged particle fluxes in the latter. All this gives indications about the possibility of the enhancement of the plasma instabilities in the regions where the magnetotail plasma interacts with the LS, which underscores the importance of investigation of the wave processes in these regions.

For electrostatic perturbations, the case of $w_{Bi} \ll w \ll w_{Be}$ corresponds to the so-called lowerhybrid waves [25]. Let us study a possibility of excitation of lower-hybrid turbulence in dusty plasmas at the Moon.

The linear dispersion relationship for the hydrodynamical instability resulting in excitation of the lower-hybrid waves under the above conditions is

$$1 + \frac{\omega_{peM}^2 + \omega_{pe(ph)}^2}{\omega_{Be}^2}\sin^2\Theta - \frac{\omega_{piM}^2 + \omega_{peM}^2\cos^2\Theta}{\omega^2} - \frac{\omega_{pd}^2}{(\omega - \mathbf{k}\cdot\mathbf{u_{pd}})^2}$$
$$- \frac{\omega_{pe(ph)}^2\cos^2\Theta}{(\omega - k_{\|}u_{dp\|})^2} = 0. \qquad (11)$$

In the earlier equation $\mathbf{k}$ is the wave vector, $\omega$ is the frequency, $\omega_{peM}$ is the plasma frequency of magnetosphere electrons, $\omega_{pe(ph)}$ is the plasma frequency of photoelectrons, $\omega_{pd}$ is the dust plasma frequency, $\omega_{piM}$ is the plasma frequency of magnetosphere ions, $\omega_{Be}$ is the electron gyrofrequency, $\mathbf{u_{pd}}$ denotes the relative speed of the near-Moon dusty plasma and the magnetosphere plasma, the subscript $\|$ representes the wave vector component parallel to the external magnetic field, $\cos\Theta = k_{\|}/|\mathbf{k}|$.

Consider the situation inherent for the lower-hybrid waves [25] when $\cos\Theta \ll 1$ and $|k_{\|}u_{d\|}| \ll |\mathbf{k}\cdot\mathbf{u_{pd}}|$. In this situation, as it will be shown below, the solution of Eq. (11) is $\omega \approx \mathbf{k}\cdot\mathbf{u_{pd}}$. Thus for typical dusty plasma parameters dispersion relation (11) can be written as

$$1 + \frac{\omega_{pe(ph)}^2}{\omega_{Be}^2} - \frac{\omega_{piM}^2 + \omega_{pe(ph)}^2\cos^2\Theta}{\omega^2} - \frac{\omega_{pd}^2}{(\omega - \mathbf{k}\cdot\mathbf{u_{pd}})^2} = 0, \qquad (12)$$

dispersion relation (12) has unstable solution with the growth rate

$$\gamma_{max}^{Hydro} = \frac{\sqrt{3}}{2^{4/3}}\frac{\omega_{pd}}{\sqrt{1 + \frac{\omega_{pe(ph)}^2}{\omega_{Be}^2}}}\left(\frac{\omega_{piM}^2 + \omega_{pe(ph)}^2\cos^2\Theta}{\omega_{pd}^2}\right)^{1/6}. \qquad (13)$$

The typical instability growth time $\tau = \left(\gamma_{max}^{Hydro}\right)^{-1}$ is about 30 s. The lower-hybrid turbulence can be generated wherever the Earth's magnetotail interacts with the near-surface dusty plasmas at the Moon in contrast to the situation with ion-acoustic turbulence [26] which is excited only in the transient and/or boundary magnetospheric layers. The time of the interaction of the near-Moon dusty plasma with the Earth's magnetosphere is much larger than $\tau$. Thus we can expect efficient nonlinear processes associated with the lower-hybrid waves.

The effective collision frequency, which determines the loss of ion momentum as a consequence of the interaction between magnetotail ions and lower-hybrid waves, is determined by

$$\nu_{eff} \sim \frac{\omega_{pd}^{2/3}\omega_{LH}^{1/3}(\cos\Theta)}{\left(1 + \omega_{pe(ph)}^2/\omega_{Be}^2\right)^{1/3}}\frac{v_{TiM}^2}{u_{pd}^2}. \qquad (14)$$

where $v_{TiM}$ is the magnetosphere ion thermal velocity.

Using the condition $eE \sim \nu_{eff} m_i u_{pd}$, we find that the characteristic electric field strength $E$ arising in the dusty plasma due to the development of lower-hybrid turbulence can be written as

$$E \sim \frac{\omega_{pd}^{2/3}\omega_{LH}^{1/3}(\cos\Theta)}{\left(1+\omega_{pe(ph)}^2/\omega_{Be}^2\right)^{1/3}}\frac{T_{iM}}{eu_{pd}}. \tag{15}$$

Calculations by formula (15) for the conditions corresponding to the magnetotail ($|\mathbf{B}| \sim 10^{-4}$ G, $\cos\Theta \sim \omega_{piM}/\omega_{pe(ph)}$, $a \sim 100$ nm, $u_{pd} \sim 1$km/s, $|Z_d| \sim 10$, $n_d \sim 10$ cm$^{-3}$, $n_{iM} \sim 10$ cm$^{-3}$, $n_{e(ph)} \sim 10^2$cm$^{-3}$, $T_{iM} \sim 100$ eV and $m_d \sim 10^{-14}$ g) show that electric fields with an amplitude of $E \sim 0.1$ V/m can be induced in the dusty plasma system near the Moon in the presence of lower-hybrid turbulence. This value is somewhat lower that the field $E \sim 1$ V/m [10], excited near the lunar surface due to its charging in the interaction with solar radiation. Nevertheless, the electric fields excited due to the development of lower-hybrid turbulence can affect the electric field pattern above the lunar surface, because the electric field arising due to the interaction of solar radiation with the LS decreases with increasing altitude.

Lower-hybrid turbulence can be significant when describing the effect of the linear dusty plasma under the interaction of the dusty plasma above the lunar surface with Earth's magnetotail on the reconnection of magnetic field line. The plasma turbulence occurring in the region where the Moon and Earth's magnetotail can be interacted. In this region, the magnetic reconnection can happen because of the Earth's magnetosphere is not restricted but also extended to tens or a hundred of Earth radii ($R_E$) [27]. Therefore, the character of the magnetic reconnection can be effected due to the interaction of the Moon with the Earth's magnetoshere.

The model of Parker-Sweet diffusion [28, 29] has been used extensively to study the reconnection, this model is modified to allow the anomalous dissipation. According to this model, the reconnection is considered as a result of mutual diffusion of magnetic fields in the Earth's magnetosphere, which are assumed to be opposite directed. The width of the transient layer of the reconnection zone is determined by the following expression:

$$d \approx \frac{c}{\omega_{pe}}\left(\frac{L\nu_{eff}}{v_A}\right)^{\frac{1}{2}}, \tag{16}$$

where $c$ is the speed of light, $v_A$ is the Alfven velocity and $L$ is the characteristic inhomogeneity scale along the direction of the transient layer. The magnitude of $L$ is typically from $5R_E$ to $10R_E$. For the lower-hybrid turbulence developed under the conditions corresponding to the magnetotail ($|B| \sim 10^{-4}$ G, $\cos\Theta \sim \omega_{piM}/\omega_{pe(ph)}$, $n_d \sim 10$ cm$^{-3}$, $n_{iM} \sim 10$ cm$^{-3}$, $u_{pd} \sim 1$km/s, $a \sim 100$ nm, $|Z_d| \sim 10$, $m_d \sim 10^{-14}$ g, $T_{iM} \sim 100$ eV, and $n_{e(ph)} \sim 10^2$cm$^{-3}$) assuming that $L = 10R_E$, we find $d \sim 500$ km. This shows that lower-hybrid wave turbulence in the vicinity of the Moon may contribute to enhancement the magnetic field reconnection process in Earth's magnetosphere.

## The $4^{th}$ chapter

In this chapter, nonlinear equations such as the modified Kadomtsev-Petviashvili equation and the modified Zakharov-Kuznetsov equation characterizing nonlinear dynamics of wave structures in dusty plasma above SSM are derived and analyzed.

**In section 4.1**, the gyrofrequency of dust particles $\omega_{Bd}$ is very small ($\omega_{Bd} \sim 10^{-8}\ \mathrm{s}^{-1}$) that we can assume that for frequencies of dust acoustic waves $\omega$ applies the relation $\omega \gg \omega_{Bd}$. In this case, the influence of the magnetic field can be neglected, but on the other hand, there is an anisotropy associated with the magnetic field vector that can affect the structure of the nonlinear wave. So, for example, if there is an almost one-dimensional wave packet in which localization along the magnetic field vector is much stronger than in other directions, a two-dimensional description similar to that which leads to the well-known Kadomtsev-Petviashvili equation. For a system of equations for a two-dimensional (in spatial coordinates) description of dusty acoustic nonlinear wave structures in a dusty plasma above SSM, charged dust grains are described hydrodynamically using the continuity equation and the equation of motion (Euler),

$$\partial_t n_d + \partial_x (n_d u_{d,x}) + \partial_y (n_d u_{d,y}) = 0, \tag{17}$$

$$\partial_t u_{d,x} + u_{d,x}\partial_x u_{d,x} + u_{d,y}\partial_y u_{d,x} = -\frac{Z_d e}{m_d}\,\partial_x\varphi, \tag{18}$$

$$\partial_t u_{d,y} + u_{d,y}\partial_y u_{d,y} + u_{d,x}\partial_x u_{d,y} = -\frac{Z_d e}{m_d}\,\partial_y\varphi, \tag{19}$$

where $\varphi$ is the self-consistent electrostatic potential in the plasma, the coordinates $x$ and $y$ are horizontal, $u_{d(x,y)}$ is the velocity of the dust particle. The equations (17), (18), (19) are valid when the soliton propagation is horizontal and, in addition, the heights $h$, at which the soliton propagation is considered, significantly exceed the Debye radius plasma photoelectrons $\lambda_{De} = \sqrt{T_e/4\pi n_e e^2}$. Here $n_e$ is the concentration of photoelectrons, $T_e$ is the temperature of photoelectrons expressed in energy units. the Gurevich formula should be used for the distribution of electrons

$$n_e = n_{e0}\left[\left(1 - \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{e\varphi/T_e}} \exp\{(-v^2)\}dv\right)\exp\left\{\left(\frac{e\varphi}{T_e}\right)\right\} + \frac{2}{\sqrt{\pi}}\sqrt{\frac{e\varphi}{T_e}}\right]. \tag{20}$$

The system of equations (17), (18), (19), and (20) are supplemented by Poisson's equation for the electrostatic potential:

$$\partial_x^2\varphi + \partial_y^2\varphi = 4\pi e\,(n_e - Z_d n_d)\,. \tag{21}$$

The derivation of the nonlinear equation in the situation under consideration is carried out according to the perturbation theory by the standard method [30, 31] of expansion in the small parameter $\varepsilon$, which uses the derivation of the asymptotic representation based on the classical analysis of dimensions. A nonlinear equation obtained in this way for dust acoustic disturbances above SSM has the form:

$$\partial_\xi \left( \partial_\tau \varphi + (c_{ds}/\sqrt{\pi}) \sqrt{e\varphi/T_e} \partial_\xi \varphi + \left( c_{ds} \lambda_{De}^2/2 \right) \partial_\xi^3 \varphi \right) = - \left( c_{ds}/2 \right) \partial_\eta^2 \varphi. \tag{22}$$

where $\varphi_1 \to \varphi$. The equation (22) is a modified Kadomtsev-Petviashvili equation, which differs from the ordinary equation in that the second (nonlinear) term of its left-hand side contains the factor $\sqrt{\varphi}$. The equation (22) has a one-dimensional solitons solution as:

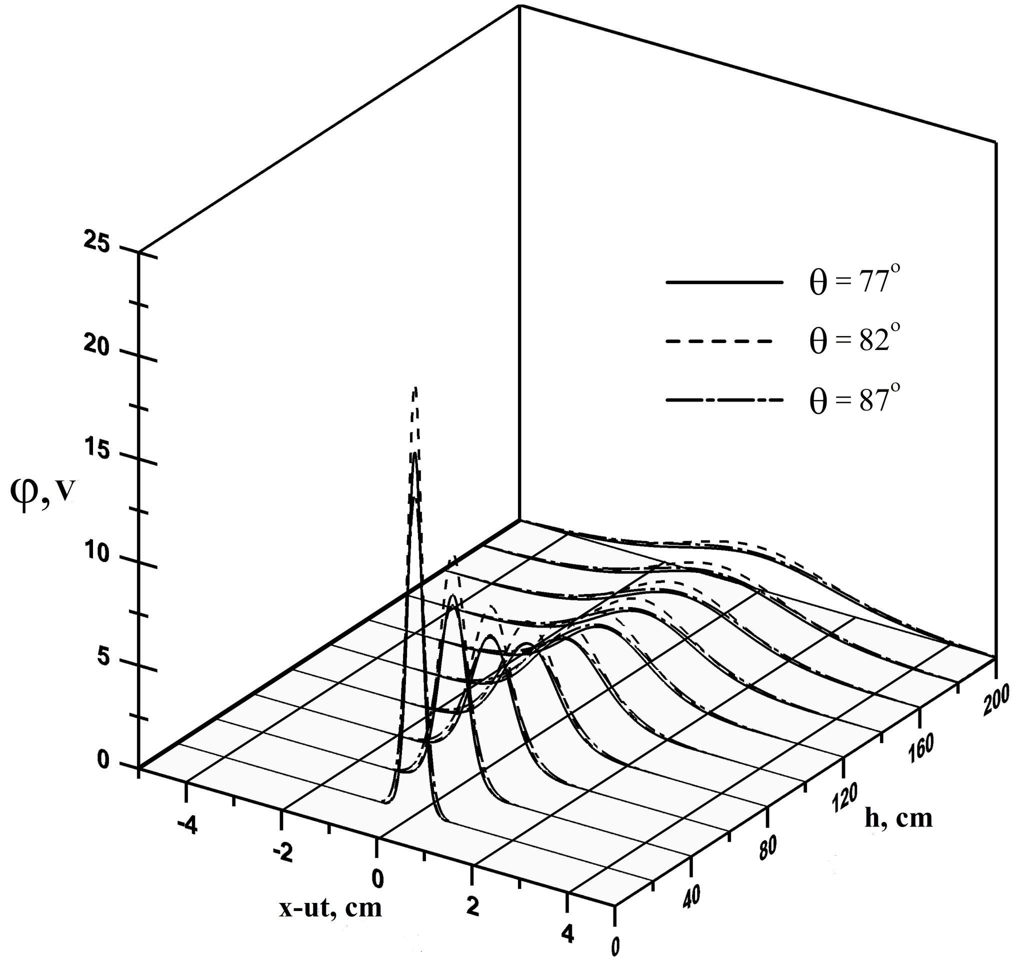



**Figure 4:** *Characteristic soliton solutions (23) for plasma parameters corresponding to heights* $h = 20$, *40, 60, 80, 100, 120, 140, 160, 180 and 200 cm and velocities* $u = 20$ *cm/s for all profiles.*

$$\varphi = \left( \frac{15u}{8a} \right)^2 \frac{1}{\cosh^4 \left[ \sqrt{u/16b} (\xi - u\tau) \right]}, \tag{23}$$

where $a = (c_{ds}/\sqrt{\pi})\sqrt{e/T_e}$, $b = c_{ds}\lambda_{De}^2/2$, $u$ –soliton propagation speed.

Figure 4 illustrates the characteristic of the soliton solutions (23) for plasma parameters corresponding to different heights $h$ and the same velocity $u = 20$ cm/s along the Moon's surface for three values of the Sun's angular height $\theta = 77^0$, $82^0$ and $87^0$. To study the stability of soliton solutions in two-dimensional space, one should linearize the equation (22) with respect to small perturbations $\delta\varphi$ of the exact solution (23). As a result, the one-dimensional soliton (23) is stable.

**In section 4.2**, we consider the opposite situation, when the frequencies of dusty acoustic waves do not exceed $\omega_{Bd}$. In an ordinary plasma, where low frequencies and a pancake-shaped wave packet in the direction along an external magnetic field, nonlinear waves are described by the well-known Zakharov–Kuznetsov equation and the magnetic field are presented in the following mathematical model.

$$\frac{\partial n_d}{\partial t} + \bigtriangledown . (n_d u_d) = 0, \tag{24}$$

$$\frac{\partial u_d}{\partial t} + (u_d . \bigtriangledown) u_d + \frac{q_d}{m_d} \bigtriangledown \phi = \frac{-Z_d e}{m_d c} (u_d \wedge B), \tag{25}$$

where $B = |\mathbf{B}|$ and in this case $u_d = u_{dx}\hat{x} + u_{dy}\hat{y} + u_{dz}\hat{z}$ are the velocity components of dust particles.

The Poisson equation has the following form:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 4\pi \left( e n_e - q_d n_d \right). \tag{26}$$

After use the standard method of expansion in small parameter, the nonlinear equation is obtained

$$\frac{\partial \phi}{\partial t} + c_{ds} \sqrt{\frac{e\phi}{\pi T_e}} \frac{\partial \phi}{\partial z} + \frac{c_{ds} \lambda_{De}^2}{2} \frac{\partial^3 \phi}{\partial z^3}$$
$$+ \frac{c_{ds} \lambda_{De}^2}{2} \frac{(\omega_{pd}^2 + \omega_{Bd}^2)}{\omega_{Bd}^2} \frac{\partial}{\partial z} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi = 0, \tag{27}$$

equation (27) is the modified Zakharov–Kuznetsov equation.

To find the solution of the modified ZK equation (27) in the form of dust acoustic waves horizontally propagating at heights $h$ that considerably exceed the Debye radius $\lambda_{De}$, we go to the coordinate system, in which the $\acute{z}$-axis is oriented along the direction of the wave packet propagation. Introducing the $\vartheta$ angle between the $\acute{z}$-direction and the magnetic field $\mathbf{B}$, we go to the new coordinate system in accordance with the following change of variables:

$$\acute{x} = x \cos\vartheta - z \sin\vartheta, \tag{28}$$

$$\acute{y} = y, \tag{29}$$

$$\acute{z} = x \sin\vartheta + z \cos\vartheta, \tag{30}$$

under these changes in the independent variables, the modified ZK equation (27) takes the following form:

$$\frac{\partial\phi}{\partial t}+\gamma_1\sqrt{\phi}\frac{\partial\phi}{\partial \acute{z}}+\gamma_2\frac{\partial^3\phi}{\partial \acute{z}^3}+\gamma_3\sqrt{\phi}\frac{\partial\phi}{\partial \acute{x}}+\gamma_4\frac{\partial^3\phi}{\partial \acute{x}^3}$$
$$+\gamma_5\frac{\partial^3\phi}{\partial \acute{x}\partial \acute{z}^2}+\gamma_6\frac{\partial^3\phi}{\partial \acute{z}\partial \acute{x}^2}+\gamma_7\frac{\partial^3\phi}{\partial \acute{z}\partial \acute{y}^2}+\gamma_8\frac{\partial^3\phi}{\partial \acute{x}\partial \acute{y}^2}=0, \quad (31)$$

where

$$\begin{aligned}
\gamma_1 &= c_{ds}\sqrt{\frac{e}{\pi T_e}}\cos\vartheta,\\
\gamma_2 &= \frac{c_{ds}\lambda_{De}^2}{2}\cos^3\vartheta+\frac{c_{ds}\lambda_{De}^2}{2}\frac{(\omega_{pd}^2+\omega_{Bd}^2)}{\omega_{Bd}^2}\sin^2\vartheta\cos\vartheta,\\
\gamma_3 &= -c_{ds}\sqrt{\frac{e}{\pi T_e}}\sin\vartheta,\\
\gamma_4 &= -\frac{c_{ds}\lambda_{De}^2}{2}\sin^3\vartheta+\frac{c_{ds}\lambda_{De}^2}{2}\frac{(\omega_{pd}^2+\omega_{Bd}^2)}{\omega_{Bd}^2}\cos^2\vartheta\sin\vartheta,\\
\gamma_5 &= \frac{c_{ds}\lambda_{De}^2}{2}\frac{(\omega_{pd}^2+\omega_{Bd}^2)}{\omega_{Bd}^2}(2\cos^2\vartheta\sin\vartheta-\sin^3\vartheta)\\
&- \frac{3c_{ds}\lambda_{De}^2}{2}\cos\vartheta^2\sin\vartheta,\\
\gamma_6 &= \frac{c_{ds}\lambda_{De}^2}{2}\frac{(\omega_{pd}^2+\omega_{Bd}^2)}{\omega_{Bd}^2}(\cos^3\vartheta-2\cos\vartheta\sin^2\vartheta)\\
&+ \frac{3c_{ds}\lambda_{De}^2}{2}\cos\vartheta\sin^2\vartheta,\\
\gamma_7 &= \frac{c_{ds}\lambda_{De}^2}{2}\frac{(\omega_{pd}^2+\omega_{Bd}^2)}{\omega_{Bd}^2}\cos\vartheta,\gamma_8=-\frac{c_{ds}\lambda_{De}^2}{2}\frac{(\omega_{pd}^2+\omega_{Bd}^2)}{\omega_{Bd}^2}\sin\vartheta.
\end{aligned}$$

The steady state one-dimensional solitons solution propagating along the $\widetilde{Z}$-axis at the velocity $u_o$ of the modified ZK equation (27) is obtained in the following form:

$$\phi=\phi_{sol}(\tilde{Z}),\ \ \widetilde{Z}=\acute{z}-u_{sol}t, \quad (32)$$

using the transformation in equation (32) , the reduced modified ZK equation in a steady state leads to

$$-u_{sol}\frac{\partial\phi_{sol}}{\partial\widetilde{Z}}+\gamma_1\sqrt{\phi_{sol}}\frac{\partial\phi_{sol}}{\partial\widetilde{Z}}+\gamma_2\frac{\partial\phi_{sol}^3}{\partial\widetilde{Z}^3}, \quad (33)$$

we impose the appropriate boundary conditions that,$\phi_{sol}\to 0$ , $\frac{d\phi_{sol}}{d\widetilde{Z}}\to 0$ and $\frac{d^2\phi_{sol}}{d\widetilde{Z}}\to 0$ as $\widetilde{Z}\to\pm\infty$, the following solitary wave solution of the equation (33) is obtained

$$\phi_{sol} = \Big(\frac{15u_{sol}}{8\gamma_1}\Big)^2 \,\mathrm{ch}^{-4}\Big(\frac{1}{2}\sqrt{\frac{\mathrm{u_{sol}}}{4\gamma_2}}(\acute{\mathrm{z}} - \mathrm{u_{sol}t})\Big). \tag{34}$$

The amplitude of soliton $\phi_o = \Big(\frac{15u_{sol}}{8\gamma_1}\Big)^2$ in equation (34) is positive, i.e. the assumption is justified that takes into account the adiabatic capture of electrons by the potential well formed by the dust-acoustic soliton.

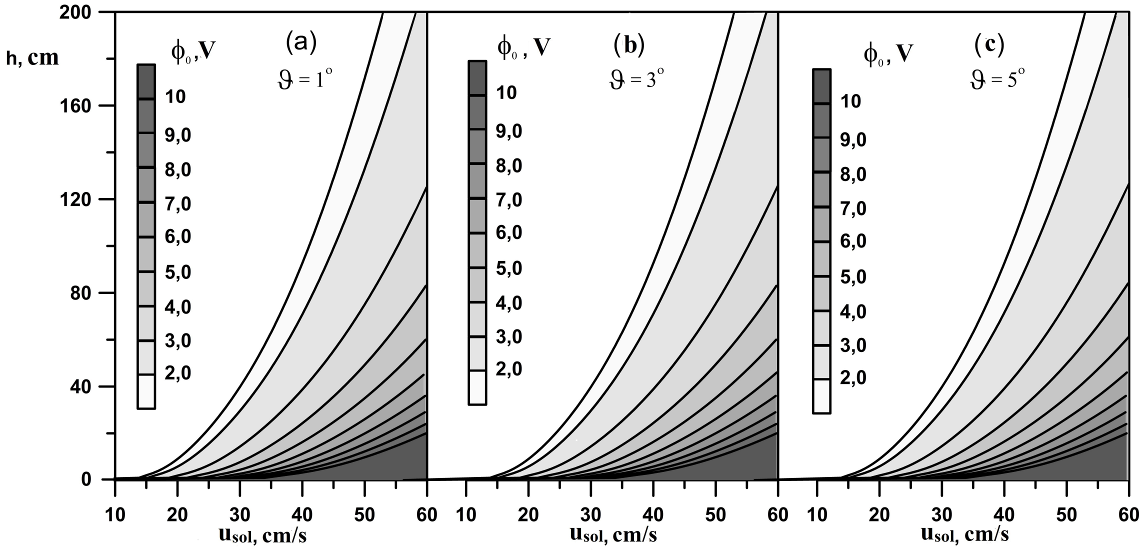



**Figure 5:** *Amplitudes of soliton solutions as functions of height $h$ above lunar surface and velocity $u_{sol}$ of soliton propagation for (a) $\vartheta = 1°$, (b) $3°$ and (c) $5°$.*

The amplitudes of soliton solutions (34) as functions of the height $h$ above the lunar surface and velocity $u_{sol}$ of the soliton propagation are shown in Fig. 5 for different angles $\vartheta$: $\vartheta = 1,\ 3$ and $5°$ correspond to Figs. 5a, 5b, and 5c, respectively. All calculations were performed for the parameters of dusty exosphere above the lunar surface corresponding to the solar declination angle $\theta$ is $82°$.

For studying the stability of soliton solutions, we use the standard method [32] of linearization of Eq. (27) with respect to small perturbations.

$$\phi = \phi_{sol}(\widetilde{Z}) + \delta\phi(\acute{x}, \acute{y}, \acute{\acute{z}}, t) \tag{35}$$

where $\phi_{sol}$ is defined by equation (34). One can obtain the following dispersion relation

$$\omega_1 = \Omega - u_{sol} l_{\acute{\acute{z}}} + \sqrt{\Omega^2 - \Lambda}, \tag{36}$$

where

$$\Omega = \frac{16}{21}\Big(\sqrt{\frac{15u_{sol}}{8\gamma_2}}\upsilon_1 - \frac{3\upsilon_2 u_{sol}}{16\gamma_2}\Big), \tag{37}$$

$$\Lambda \;=\; \frac{512}{945}\Big((\frac{15u_{sol}}{8\gamma_1})^4\upsilon_1^2 - (\frac{75u_{sol}^2}{128\gamma_1\gamma_2})\upsilon_1\upsilon_2$$

$$- \frac{15}{512}\frac{u_{sol}^2 \upsilon_2^2}{\gamma_2^2} + \frac{15}{128}\frac{u_{sol}^2 \upsilon_3}{\gamma_2}\Big), \tag{38}$$

It can be seen from dispersion relation (36) that in the case of $(\Lambda < \Omega^2)$, the dispersion law is real and the solution is stable. Otherwise, the instability occurs at $(\Lambda > \Omega^2)$. The analysis of Eq. (36) leads to the following threshold value $\omega_{Be,th}$ for the instability development

$$\omega_{Be,th}^2 = \frac{1 + \frac{l_\zeta^2}{l_\eta^2}(1 - \frac{9}{7}\tan^2\vartheta)}{\sin^2\vartheta - \frac{9 l_\zeta^2}{7 l_\eta^2}\tan^2\vartheta}, \tag{39}$$

the instability growth rate is as follows

$$\Gamma = \frac{2}{\sqrt{63}}\frac{u_{sol}[(1+\omega_{Be}^2)P_i]^{1/2}}{(\omega_{Be}^2 + \sin^2\vartheta)}, \tag{40}$$

where

$$P_i = \left[1 + \frac{l_\zeta^2}{l_\eta^2}(1 - \frac{9}{7}\tan^2\vartheta)\right]\omega_{Be}^2 + \left[\sin^2\vartheta - \frac{9 l_\zeta^2}{7 l_\eta^2}\tan^2\vartheta\right]. \tag{41}$$

# Conclusions

The main results of the dissertation work are concluded as follows:

1. Investigation of the influence of magnetic fields of the Earth's magnetotail and of the magnetic fields in the regions of magnetic anomalies of the Moon on the dust transport and on the formation of dusty plasmas above the Moon has been performed. The research had shown a possibility of the existence of positively charged dust for the whole range of the lunar latitudes (from $-90^o$ to $90^o$). Consequently, dusty plasmas above the sunlit part of the Moon can exist in the entire range of lunar latitudes. Dust grain transport from the region of high lunar latitudes toward the equator of the Moon has been demonstrated to be due to the uncompensated magnetic part of the Lorentz force, which is due to the magnetic fields of the Earth's magnetotail. It provides a new qualitative effect that does not exist in the absence of the magnetic field. The magnetic component of the Lorenz force acting on the dust grain in the fields of magnetic anomalies is either lower or comparable to the similar force calculated for the magnetic field of the Earth's magnetotail in lunar orbit. Nevertheless, due to the existing localization of the regions of magnetic anomalies, these regions can lead only to some changes in dust grain trajectories, deflecting them from those formed as a result of the influence of magnetic fields of the Earth's magnetotail, the general trend of the motion of dust grains remaining the same as in the case of the dust grain motion under the action of the magnetic fields of the Earth's magnetotail.

2. The investigation of the dust grain trajectories above the lunar surface has shown that the processes of variation in the charges of dust grains, leading to the attenuation of their oscillations over the lunar surface, are too fast in comparison with the day duration on the Moon. In this connection, most of the dusts in the dusty plasmas above the sunlit part of the lunar surface can be considered as levitated. Only very fine grains (with sizes smaller than several dozens of $\mu$m) do not go into levitating grain mode during the entire day on the Moon and can be treated as lofted.

3. An analysis of linear and nonlinear processes related to the presence of magnetic fields, which can be important in dusty plasmas at the Moon has been performed. It has been shown the significance of lower-hybrid turbulence excitation which is due to the interaction of the Earth's magnetotail with dusty plasma near the lunar surface. The growth rates characterizing the excitation of the lower-hybrid turbulence have been found. It has been demonstrated that in contrast to the situation with ion-acoustic turbulence, which is excited only in the transient and/or boundary magnetospheric layers, the lower-hybrid turbulence can be generated wherever the Earth's magnetotail interacts with the near-surface dusty plasmas at the Moon. The effective collision frequency, which determines the loss of ion momentum as a consequence of the interaction between magnetotail ions and lower-hybrid waves, has been determined, and the typical electric fields, which appear due to lower-hybrid turbulence development, have been estimated.

4. Nonlinear equations such as the modified Kadomtsev-Petviashvili equation and the modified Zakharov-Kuznetsov equation characterizing nonlinear dynamics of wave structures in dusty plasma above the Moon's illuminated side have been derived and analyzed. The modified Kadomtsev– Petviashvili equation describes nonlinear dynamics of nearly one-dimensional wave structures in dusty plasma above the Moon's lighted side in the situation in which localization along magnetic-field vector is much stronger than along other directions. The equation differs from the ordinary Kadomtsev-Petviashvili equation by the nonlinear term being non-analytical. Modified Kadomtsev–Petviashvili differs from generalizations of the Kadomtsev–Petviashvili equation in which nonlinearity retains the same form as in the ordinary Kadomtsev–Petviashvili equation but higher-order corrections for dispersion are taken into account. An analytical expression governing one-dimensional soliton solution to the modified Kadomtsev–Petviashvili equation has been obtained. The solution differs from the well-known one-dimensional soliton solutions to the Korteweg–De Vries and ordinary Kadomtsev–Petviashvili equations. Stability analysis of the one-dimensional soliton solution showed that it is stable. The modified Zakharov-Kuznetsov equation is the nonlinear equation that describes the dynamics of nonlinear wave structures in the dusty plasma above the illuminated surface of the Moon in the case of low frequencies and pancake-like shape of wave packet in the direction along the external magnetic field. The analytical formula for the one-dimensional soliton solution of the modified Zakharov-Kuznetsov equation has been derived. Possible applications of the obtained soliton solutions are related to

the description of the so-called transient lunar phenomena representing short-lived light, changes in color or appearance on the surface of the Moon.

- **List of publications and conferences on the theme of the thesis**


1. Popel, S. I., Kassem, A. I., Izvekova, Y. N., and Zelenyi, L. M. Lower-hybrid turbulence in the near-surface lunar dusty plasmas. *Physics Letters A*, VOL 384(26), 126627, 2020.

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