# Large-scale structures in a turbulent fluid with solid particles and with gas bubbles

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# Outline

- Introduction
- Mean field hydrodynamic
- Fluid with solid particles
- Fluid with gas bubbles
- Conclusions

# Magnetic field and vorticity equations

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \Delta \mathbf{B}$$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) + \eta \Delta \boldsymbol{\omega}$$

HK Moffat Some developments in the theory of turbulence. *J. Fluid Mech.* **106**, 27–47. 1981

# INTRODUCTION

- The formation of large scale structures is one of the most fascinating problems in the hydrodynamic turbulence theory
- The global breaking of mirror symmetry in turbulence is often considered as a probable source of coherent structures in turbulence
- Such  turbulence is helical and is characterized by the mean helicity:

# HELICAL TURBULENCE

turbulence is characterized by the mean helicity:

$$\mathcal{H} = \int \langle \vec{v} \cdot rot\vec{v} \rangle d\mathbf{V} \, .$$

Here *brackets* denote the ansamble averaging.

# Homogeneous turbulence

The homogeneity of turbulence and the incompressibility condition make it possible to reduce the common form of autocorrelational tensor to:

$$\hat{Q}^0_{ij}(\vec{k}, w) = (\delta_{ij} - \frac{k_i k_j}{k^2}) \frac{E(k,w)}{4\pi k^2} + i\varepsilon_{ijk} k_k \frac{H(k,w)}{8\pi k^4}$$

# Reynolds stresses expansion

$$R_{ij} = \left\langle u_j^0 u_j^0 \right\rangle + \langle u \rangle_l \left[ \frac{\partial \langle u_l u_j \rangle}{\partial \langle u \rangle_l} \right]_{\langle \mathbf{u} \rangle = 0} + \mathcal{O}\left( \langle \mathbf{u} \rangle^2 \right)$$

Basic flow $\mathbf{u}^0$

$\langle \mathbf{u} \rangle$ large scale perturbation

# Navier-Stokes equations for incompressible fluid

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}\nabla)\vec{V} = -\nabla P + \nu_0 \Delta \vec{V} + \vec{F},$$

$$\nabla \vec{V} = 0,$$

# Reynolds averaged equations

$$\frac{\partial \bar{\vec{V}}}{\partial t} + (\bar{\vec{V}}\nabla)\bar{\vec{V}} + < (\vec{V}^t\nabla)V^t >) = -\nabla \bar{P} + \nu_0 \Delta \bar{\vec{V}},$$

$$\nabla \bar{\vec{V}} = 0,$$

$$\frac{\partial \vec{V}^t}{\partial t} + (\vec{V}^t\nabla)\vec{V}^t - < (\vec{V}^t\nabla)\vec{V}^t > + (\vec{V}^t\nabla)\bar{\vec{V}}) + (\bar{\vec{V}}\nabla)\vec{V}^t)$$

$$= -\nabla P^t + \nu_0 \Delta \vec{V}^t + \vec{f}^t,$$

$$\nabla \vec{V}^t = 0.$$

# Equations for fluid with Solid Particles

$$\frac{\partial n}{\partial t} + div\left(n\bar{V}_S\right) = 0$$

$$div\left(\bar{V}\right) = -\frac{4}{3}\pi a^3 div\left(n\right)\left(\bar{V}_S - \bar{V}\right)$$

$$\frac{\partial \bar{V}_S}{\partial t} + \beta\left(\bar{V}_S - \bar{V}\right) + \left(\bar{V}_S \nabla\right)\bar{V}_S + \frac{1}{\rho_S}\nabla P = 0$$

$$\frac{\partial \bar{V}}{\partial t} + \gamma n\left(\bar{V}_S - \bar{V}\right) + \left(\bar{V}\nabla\right)\bar{V} + \frac{1}{\rho}\nabla P = \nu\Delta\bar{V}$$

# Flow Decomposition

$$\bar{V} = \langle \bar{V} \rangle + \bar{V}^0 + \bar{V}^1$$

$$\bar{V}_S = \langle \bar{V}_S \rangle + \bar{V}_S^0 + \bar{V}_S^1$$

$$n = \langle n \rangle + n_0 + n_1$$

$$P = \langle P \rangle + P^0 + P^1$$

$$\langle \bar{V}^0 \rangle = \langle \bar{V}^1 \rangle = 0$$

$$\langle \bar{V}_S^0 \rangle = \langle \bar{V}_S^1 \rangle = 0$$

$$\langle P^0 \rangle = \langle P^1 \rangle = 0$$

# Mean Flow Equations

$$div\left(\left\langle \bar{V}\right\rangle\right)=\frac{4}{3}\pi a^3\frac{\partial\langle n\rangle}{\partial t}+\frac{4}{3}\pi a^3 div\left(n_0\right)\left\langle \bar{V}\right\rangle+\frac{4}{3}\pi a^3 div\left(\bar{N}\right)$$

$$\frac{\partial\left\langle \bar{V}_S\right\rangle}{\partial t}+\beta\left(\left\langle \bar{V}_S\right\rangle-\left\langle \bar{V}\right\rangle\right)+Q_S+\frac{1}{\rho_S}\nabla\langle P\rangle=0$$

$$\frac{\partial\langle n\rangle}{\partial t}+div\left(n_0\left\langle \bar{V}_S\right\rangle\right)+div\left(\bar{N}_S\right)=0$$

$$\frac{\partial\left\langle \bar{V}\right\rangle}{\partial t}+\gamma\langle n\rangle\left(\left\langle \bar{V}_S\right\rangle-\left\langle \bar{V}\right\rangle\right)+\gamma\left(\bar{N}-\bar{N}_S\right)+Q+\frac{1}{\rho}\nabla\langle P\rangle=\nu\Delta\left\langle \bar{V}\right\rangle$$

# Correlations

$$\bar{Q} = \left\langle \left( \bar{V}^1 \nabla \right) \bar{V}^0 \right\rangle + \left\langle \left( \bar{V}^0 \nabla \right) \bar{V}^1 \right\rangle$$

$$\bar{Q}_S = \left\langle \left( \bar{V}_S^1 \nabla \right) \bar{V}_S^0 + \left( \bar{V}_S^0 \nabla \right) \bar{V}_S^1 \right\rangle$$

$$\bar{N} = \left\langle n^1 \bar{V}^0 \right\rangle$$

$$\bar{N}_S = \left\langle n^1 \bar{V}_S^0 \right\rangle$$

# Perturbation Equations

$$\frac{\partial n^1}{\partial t} + div\left(\left\langle n \right\rangle\right)\bar{V}_S^0 + n_0 div\left(\bar{V}_S'\right) = 0$$

$$div\left(\bar{V}^1\right) = \frac{4}{3}\pi a^3 \frac{\partial n^1}{\partial t} + \frac{4}{3}\pi a^3 div\left(\bar{V}^0 \nabla\right)\left\langle n \right\rangle$$

$$\frac{\partial\left\langle \bar{V}_S^1 \right\rangle}{\partial t} + \beta\left(\bar{V}_S^1 - \bar{V}^1\right) + \left(\left\langle \bar{V} \right\rangle \nabla\right)\bar{V}_S^0 + \left(\bar{V}_S^0 \nabla\right)\left\langle \bar{V}_S \right\rangle + \frac{1}{\rho_S}\nabla P^1 = 0$$

$$\frac{\partial \bar{V}^1}{\partial t} - \gamma n_0\left(\bar{V}^1 - \bar{V}_S^1\right) + \gamma\left\langle n \right\rangle\left(\bar{V}^0 - \bar{V}_S^0\right) + \left(\left\langle \bar{V} \right\rangle \nabla\right)\bar{V}^0 + \left(\bar{V}^0 \nabla\right)\left\langle \bar{V} \right\rangle +$$

$$+\frac{1}{\rho}\nabla P^1 = \nu \Delta \bar{V}^1$$

# Mean-Field Equations

$$\frac{\partial\langle n\rangle}{\partial t}+n_0 div\left(\langle\bar{V}_S\rangle\right)=0$$

$$div\left(\langle\bar{V}\rangle\right)=\frac{4}{3}\pi a^3\frac{\partial}{\partial t}\langle n\rangle$$

$$\frac{\partial\langle\bar{V}_S\rangle}{\partial t}+\beta\left(\langle\bar{V}_S\rangle-\langle\bar{V}\rangle\right)+\frac{1}{\rho_S}\langle P\rangle=0$$

$$\frac{\partial\langle\bar{V}\rangle}{\partial t}-\gamma n_0\left(\langle\bar{V}\rangle-\langle\bar{V}_S\rangle\right)+\alpha rot\left(\langle\bar{V}\rangle\right)+\frac{1}{\rho}\nabla P=\nu\Delta\langle\bar{V}\rangle$$

$$\gamma = \frac{2}{3} V \int \frac{k^4 A(k,\omega)\, dk\, d\omega}{V\left(\nu k^2 + \frac{\rho_S}{\rho}\right) + \gamma n_0 + \frac{\rho}{\rho_S}(\beta - i\omega)}$$

# Mean Vorticity Equations

$$\bar{\Omega} = rot\left(\left\langle \bar{V} \right\rangle\right); \qquad \bar{\Omega}_S = rot\left(\left\langle \bar{V}_S \right\rangle\right)$$

$$\frac{\partial \bar{\Omega}_S}{\partial t} + \beta\left(\bar{\Omega}_S - \bar{\Omega}\right) = 0$$

$$\frac{\partial \bar{\Omega}}{\partial t} + \gamma n_0\left(\bar{\Omega}_S - \bar{\Omega}\right) + \alpha rot\left(\left\langle \bar{V} \right\rangle\right) = \nu \Delta \bar{\Omega}$$

# Large-Scale Instability

$$-\frac{1}{\beta}(-i\omega)^2 - i\omega\left(\frac{\gamma n_0 + \nu k^2 \pm \alpha k}{\beta} + 1\right) + \left(\nu k^2 \pm \alpha k\right) = 0$$

$$-i\omega = -\frac{1}{2}\left(\gamma n_0 + \nu k^2 - \alpha k + \beta\right) \pm \sqrt{\left(\gamma n_0 + \nu k^2 + \beta - \alpha k\right)^2 - 4\beta\left(\nu k^2 - \alpha k\right)}$$

# Fluid with Gas Bubbles

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho V_i) = 0$$

$$\frac{\partial}{\partial t} V_i + (V_k \nabla_k) V_i = -\frac{\nabla p}{\rho} + \frac{\nu}{\rho} \Delta^2 V_i$$

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_o}(p_r - p)$$

# Fluid with Gas Bubbles

$$\rho = \rho_0 (1 - nU + \chi p)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho_0 V_i) = 0$$

$$\frac{\partial}{\partial t} V_i + (V_k \nabla_k) V_i = -\frac{\nabla p}{\rho_0} + \frac{\nu}{\rho_0} \Delta V_i$$

$$\ddot{U} + w_0^2 U = -\varepsilon p$$

# Mean-Field Equations

$$\frac{\partial \overline{V_i}}{\partial t} = \alpha rot \overline{V_i} - \frac{\nabla_i \overline{p}}{\rho_0} + \frac{\nu}{\rho_0} \Delta^2 \overline{V_i}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\overline{V_i} \rho_0) = 0$$

$$\overline{\rho} = \rho_0 (1 - n\overline{U} + \chi \overline{p})$$

$$\frac{d^2 \overline{U}}{dt^2} + w_0^2 \overline{U} = -\varepsilon \overline{p}$$

# Mean-Field Equations

$$\frac{\partial \overline{V_i}}{\partial t} = \alpha rot \overline{V_i} - \frac{\nabla_i \overline{p}}{\rho_0} + \frac{\nu}{\rho_0} \Delta^2 \overline{V_i}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\overline{V_i} \rho_0) = 0$$

$$\overline{\rho} = \rho_0 (1 - n\overline{U} + \chi \overline{p})$$

$$\frac{d^2 \overline{U}}{dt^2} + w_0^2 \overline{U} = -\varepsilon \overline{p}$$

$$\alpha = \frac{8\pi}{3}\int_{-\infty}^{+\infty} \frac{K^2\left(\chi + \frac{n\varepsilon}{w^2 + w_0^2}\right)A(K,w)dKdw}{\left(w - \frac{\nu}{\rho_0}K^2\right)\left(\chi + \frac{n\varepsilon}{w^2 + w_0^2}\right) - K^2}$$

# Mean-Field Equations

$$\frac{\partial \overline{V_i}}{\partial t} = \alpha rot \overline{V_i} - \frac{\nabla_i \overline{p}}{\rho_0} + \frac{\nu}{\rho_0} \Delta^2 \overline{V_i}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\overline{V_i} \rho_0) = 0$$

$$\overline{\rho} = \rho_0 (1 - n\overline{U} + \chi \overline{p})$$

$$\frac{d^2 \overline{U}}{dt^2} + w_0^2 \overline{U} = -\varepsilon \overline{p}$$

# Vorticity Equations

$$\frac{\partial}{\partial t}\bar{\Omega} + \alpha rot\bar{\Omega} = \nu' \Delta \Omega$$

$$\left(w - i\nu' K^2\right)^2 = -\alpha^2 K^2$$

# Conclusions

- Mean-field hydrodynamics for multi-phase flows
- Large scale instability for flows with suspended solid particles
- Large Scale instability for flows with oscillating gas bubles