Волны и зональные течения в плазменной астрофизике

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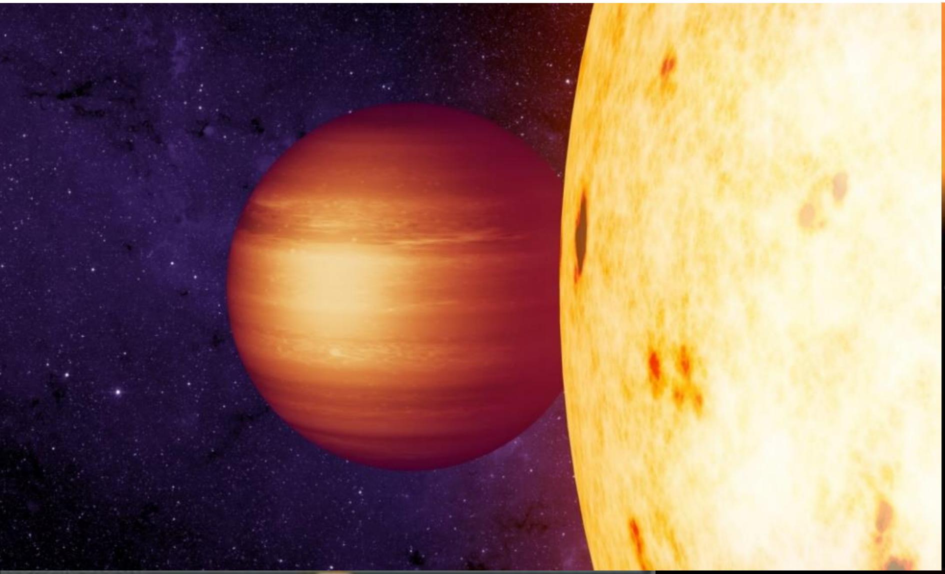
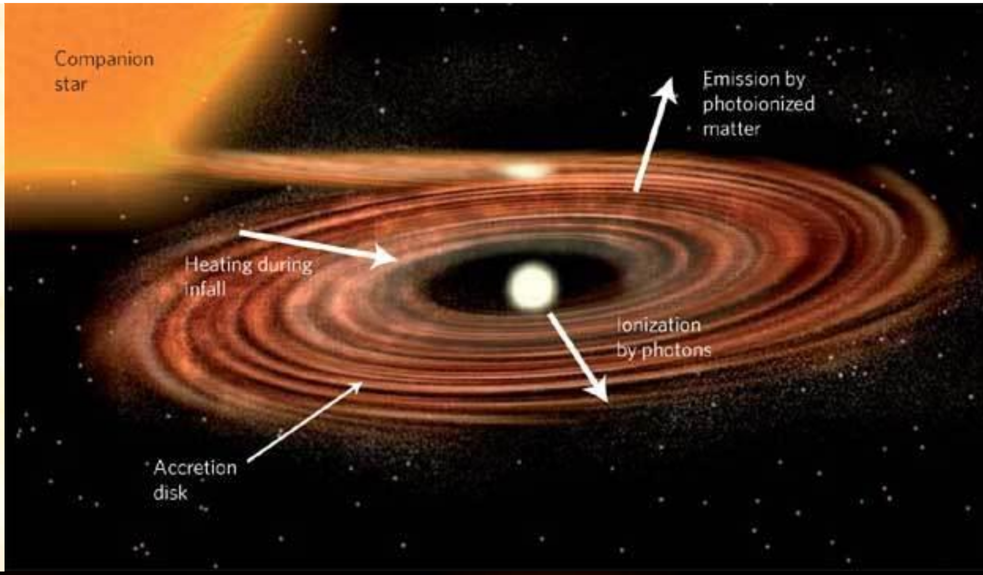
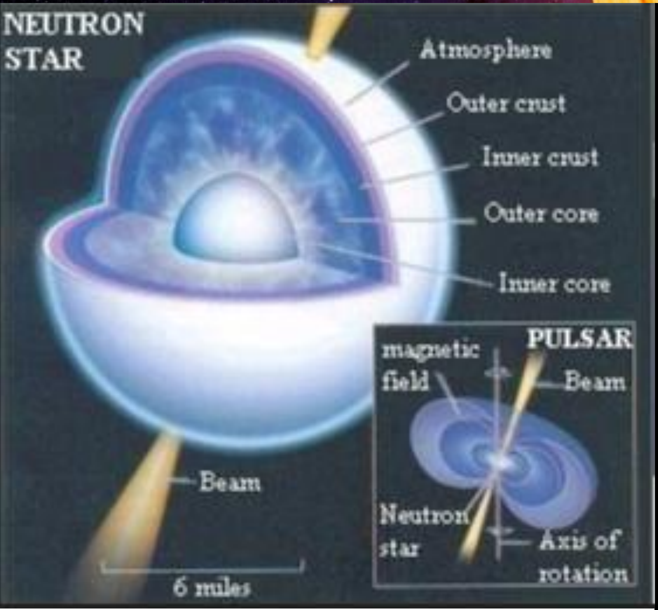
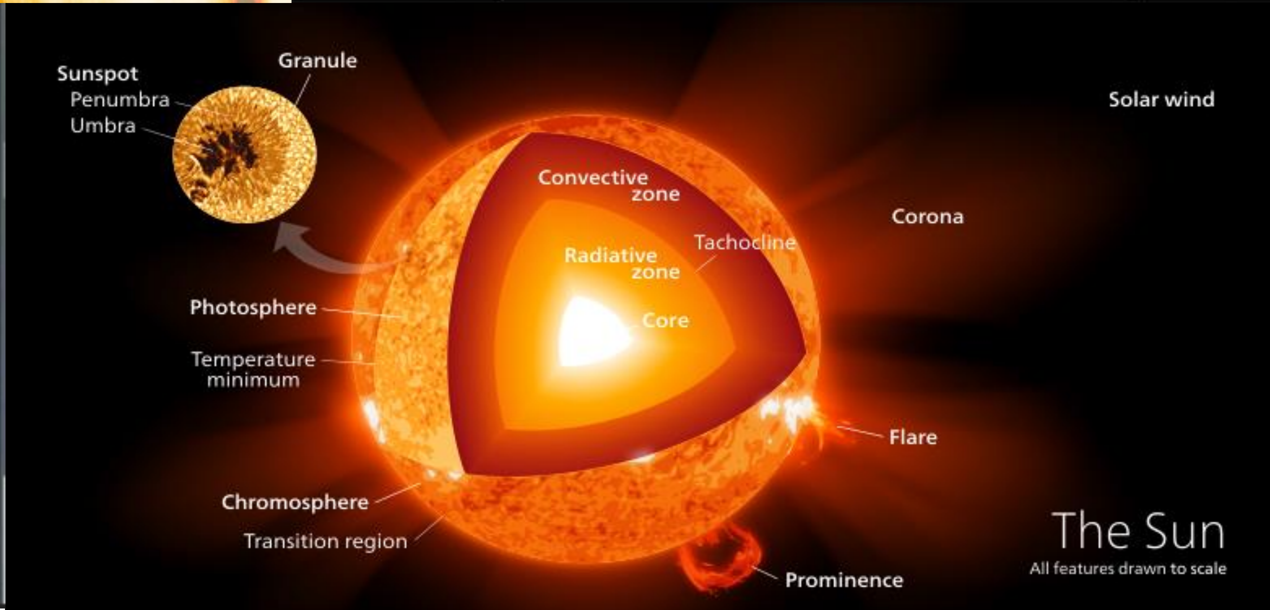
26 Ноября 2024

Plasma astrophysics









nature astronomy

LETTERS

PUBLISHED: 27 MARCH 2017 | VOLUME: 1 | ARTICLE NUMBER: 0086

The detection of Rossby-like waves on the Sun

Scott W. McIntosh[1]*, William J. Cramer[2], Manuel Pichardo Marcano[3] and Robert J. Leamon[4]

Rossby waves are a type of global-scale wave that develops in planetary atmospheres, driven by the planet's rotation[1]. They propagate westward owing to the Coriolis force, and their characterization enables more precise forecasting of weather on Earth[2,3]. Despite the massive reservoir of rotational energy available in the Sun's interior and decades of observational investigation, their solar analogue defies unambiguous identification[4–6]. Here we analyse a combined set of images obtained by the Solar TErrestrial RElations Observatory (STEREO) and the Solar Dynamics Observatory (SDO) spacecraft between 2011 and 2013 in order to follow the evolution of small bright features, called brightpoints, which are tracers of rotationally driven large-scale convection[7]. We report the detection of persistent, global-scale bands of magnetized activity on the Sun that slowly meander westward in longitude and display Rossby-wave-like behaviour. These magnetized Rossby waves allow us to make direct connections between decadal-scale solar activity and that on much shorter timescales. Monitoring the properties of these waves, and the wavenumber of the disturbances that they generate, has the potential to yield a considerable improvement in forecast capability for solar activity and related space weather phenomena.

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LETTERS

https://doi.org/10.1038/s41550-018-0460-x

Global-scale equatorial Rossby waves as an essential component of solar internal dynamics

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The Sun's complex dynamics is controlled by buoyancy and rotation in the convection zone. Large-scale flows are dominated by vertical motions[1] and appear to be weaker than expected in the solar interior[2]. One possibility is that waves of vorticity due to the Coriolis force, known as Rossby waves[3] or r modes[4], remove energy from convection at the largest scales[5]. However, the presence of these waves in the Sun is still debated. Here, we unambiguously discover and characterize retrograde-propagating vorticity waves in the shallow subsurface layers of the Sun at azimuthal wavenumbers below 15, with the dispersion relation of textbook sectoral Rossby waves. The waves have lifetimes of several months, well-defined mode frequencies below twice the solar rotational frequency, and eigenfunctions of vorticity that peak at the equator. Rossby waves have nearly as much vorticity as the convection at the same scales, thus they are an essential component of solar dynamics. We observe a transition from turbulence-like to wave-like dynamics around the Rhines scale[6] of angular wavenumber of approximately 20. This transition might provide an explanation for the puzzling deficit of kinetic energy at the largest spatial scales.

several years. We use a six-year-long time series of intensity images of the solar photosphere from the Helioseismic and Magnetic Imager (HMI) instrument aboard the Solar Dynamics Observatory (SDO) spacecraft[7]. These full-disk images are recorded at a cadence of 45 s and a spatial resolution that is high enough to resolve the photospheric granules, which are prominent convection features 1,500 km in size. Granules extend from the solar surface down to a few 100 km below the solar surface and can be used as tracers of the larger-scale horizontal flows they are embedded in. We derive the two horizontal components of the flow velocity at the solar surface by following the motions of granules between consecutive pairs of HMI intensity images every 30 min (see Methods). The radial vorticity of the flows is then computed in a frame rotating at $\Omega_{ref}/2\pi = 453.1$ nHz, which is the mean solar surface equatorial rotation rate for 2010–2016 inferred from f-mode helioseismology[8] using SDO/HMI observations.

Figure 1 shows maps of the horizontal vector velocity (arrows) and the radial component of the vorticity (background colours) for three consecutive solar rotation periods after applying a Gaussian spatial filter with standard deviation (s.d.) of 7°. These maps exhibit a complex flow pattern with many examples of vortical flow fea-

Presentation Outline

- MHD equations in shallow water approximation with external vertical magnetic field

- Beta plane approximation

- Linear theory, weakly nonlinear theory, parametric instabilities

- 2D β-plane MHD turbulence, zonal flows, anisotropic scale in MHD turbulence

MHD Shallow Water Equations

$$\frac{\partial h}{\partial t} + \frac{\partial h v_x}{\partial x} + \frac{\partial h v_y}{\partial y} = 0$$

$$\frac{\partial h v_x}{\partial t} + \frac{\partial (h v_x^2 - h B_x^2)}{\partial x} + \frac{\partial (h v_x v_y - h B_x B_y)}{\partial y} + gh\frac{\partial h}{\partial x} - f h v_y + B_0 B_x = 0$$

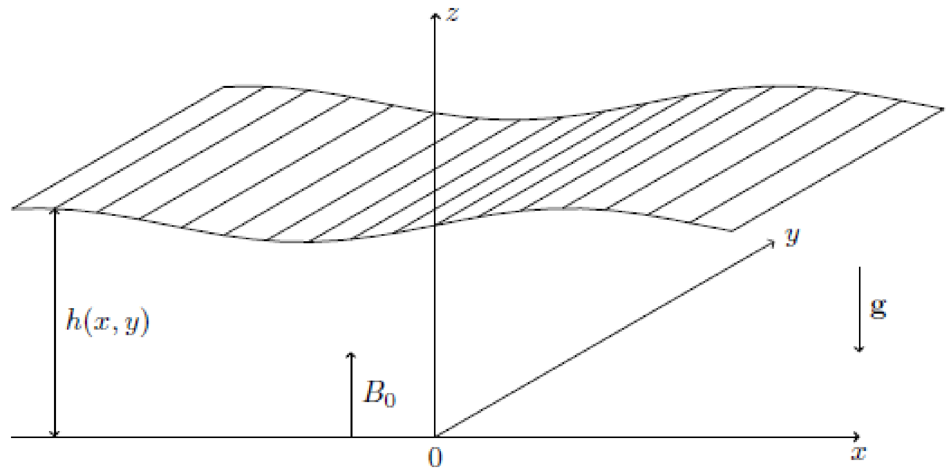
$$\frac{\partial h v_y}{\partial t} + \frac{\partial (h v_x v_y - h B_x B_y)}{\partial x} + \frac{\partial (h v_y^2 - h B_y^2)}{\partial y} + gh\frac{\partial h}{\partial y} + f h v_x + B_0 B_y = 0$$

$$\frac{\partial h B_x}{\partial t} + \frac{\partial (h v_y B_x - h v_x B_y)}{\partial y} - B_0 v_x = 0$$

$$\frac{\partial h B_y}{\partial t} + \frac{\partial (h v_x B_y - h v_y B_x)}{\partial y} - B_0 v_y = 0$$

$$\frac{\partial h B_x}{\partial x} + \frac{\partial h B_y}{\partial y} + B_z = 0$$

$$\frac{\partial B_z}{\partial t} + B_0\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = 0$$



Depth averaged velocities v_x, v_y
Depth averaged magnetic field B_x, B_y
Layer depth h

Vertical external magnetic field B_0
Coriolis parameter $f = 2\Omega$

Klimachkov D.A., Petrosyan A.S., 2017

Linear Waves. Magnetostrophic and magneto-Poincare modes

Dispersion relation:

$$\omega^5 - \omega^3\left(gHk^2 + f^2 + 2\left(\frac{B_0}{H}\right)^2\right) + \left(\frac{B_0}{H}\right)^2\left(gHk^2 + \left(\frac{B_0}{H}\right)^2\right)\omega = 0$$

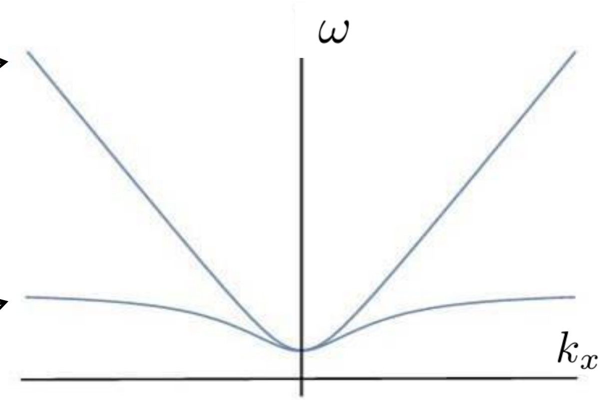
$h = H$ - stationary solution

Magneto-Poincare mode:

$$\omega^2 = \frac{gHk^2}{2} + \frac{f^2}{2} + \left(\frac{B_0}{H}\right)^2 + \frac{1}{2}\sqrt{gHk^2(gHk^2 + 2f^2) + f^2\left(f^2 + 4\left(\frac{B_0}{H}\right)^2\right)}$$

Magnetostrophic mode:

$$\omega^2 = \frac{gHk^2}{2} + \frac{f^2}{2} + \left(\frac{B_0}{H}\right)^2 - \frac{1}{2}\sqrt{gHk^2(gHk^2 + 2f^2) + f^2\left(f^2 + 4\left(\frac{B_0}{H}\right)^2\right)}$$



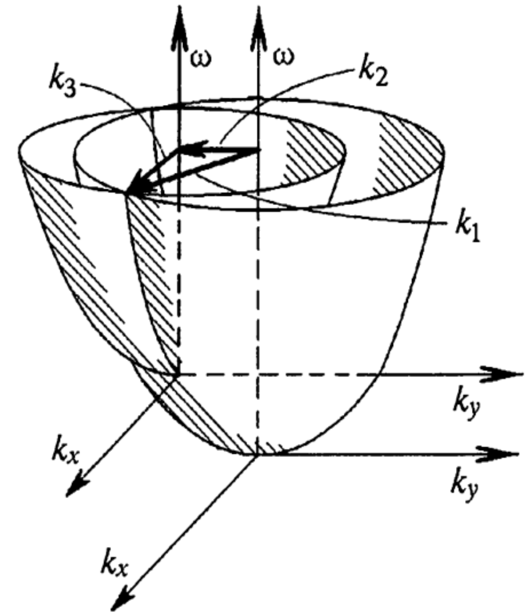
In nonrotating case where $f = 0$:

$$\omega^2 = \begin{cases} gHk^2 + \left(\frac{B_0}{H}\right)^2 & \text{- magnetogravity wave} \\ \left(\frac{B_0}{H}\right)^2 & \text{- Alfven wave} \end{cases}$$

Heng K., Spitkovsky A., 2009
Klimachkov D.A., Petrosyan A.S., 2016

Phase-matching Conditions

Possibility of three-wave interaction with isotropic $\omega(\mathbf{k})$

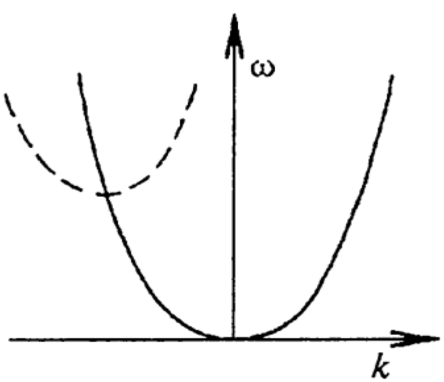


Necessary conditions

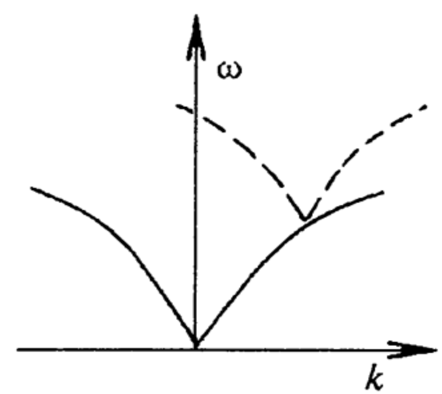
$$\omega_1 = \omega_2 + \omega_3$$

$$\mathbf{k}_1 = \mathbf{k}_2 + \mathbf{k}_3$$

Interaction is allowed:



Interaction is forbidden:



Multiscale Asymptotic Method

Solution in a form of a series in the small parameter:

$$u = \begin{pmatrix} h \\ v_x \\ v_y \\ B_x \\ B_y \end{pmatrix} = \begin{pmatrix} h_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \varepsilon \begin{pmatrix} h_1 \\ v_{x1} \\ v_{y1} \\ B_{x1} \\ B_{y1} \end{pmatrix} + \varepsilon^2 \begin{pmatrix} h_2 \\ v_{x2} \\ v_{y2} \\ B_{x2} \\ B_{y2} \end{pmatrix} = u_0 + \varepsilon u_1 + \varepsilon^2 u_2$$

u_0 - stationary solution

u_1 - linear solution

$(t, x, y) \longrightarrow$ slow (T_1, X_1, Y_1), fast (T_0, X_0, Y_0)

according to:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial T_0} + \varepsilon \frac{\partial}{\partial T_1} + \ldots$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial X_0} + \varepsilon \frac{\partial}{\partial X_1} + \ldots$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial Y_0} + \varepsilon \frac{\partial}{\partial Y_1} + \ldots$$

Terms propotional to ε^2:

$$\mathbf{A}\frac{\partial u_1}{\partial T_0} + \mathbf{B}^1 \frac{\partial u_1}{\partial X_0} + \mathbf{B}^2 \frac{\partial u_1}{\partial Y_0} + \mathbf{C}u_1 = -(\mathbf{K}\frac{\partial u_0}{\partial T_0} + \mathbf{L}^1 \frac{\partial u_0}{\partial X_0} + \mathbf{L}^2 \frac{\partial u_0}{\partial Y_0} + \mathbf{M}u_0) - (\mathbf{A}\frac{\partial u_0}{\partial T_1} + \mathbf{B}^1 \frac{\partial u_0}{\partial X_1} + \mathbf{B}^2 \frac{\partial u_0}{\partial Y_1})$$

Compatibility condition:

$$\mathbf{H}u_1 = f(u_0, u_0) + \mathbf{R}\frac{\partial u_0}{\partial \vec{r}}$$

$$(z, f(u_0, u_0) + \mathbf{R}\frac{\partial u_0}{\partial \vec{r}}) = 0$$

z – eigenvector of adjoint for **H** matrix

Equations for the Amplitudes of Interacting Waves

Equations for three magneto-Poincare waves:

$$\frac{\partial\gamma}{\partial T_1}+a^1\frac{\partial\gamma}{\partial X_1}+b^1\frac{\partial\gamma}{\partial Y_1}=f^1\alpha\beta$$

$$\frac{\partial\alpha}{\partial T_1}+a^2\frac{\partial\alpha}{\partial X_1}+b^2\frac{\partial\alpha}{\partial Y_1}=f^2\gamma\beta^*$$

$$\frac{\partial\beta}{\partial T_1}+a^3\frac{\partial\beta}{\partial X_1}+b^3\frac{\partial\beta}{\partial Y_1}=f^3\gamma\alpha^*$$

α,β,γ - amplitudes of interacting magneto-Poincare waves

(T_1,X_1,Y_1) - “slow” variables

a^i, b^i, f^i - constants are determined by the initial conditions $(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k}_3, f_0,\mathrm{g},H,B_0)$ are different for every case of three-waves interactions:

- Three magneto-Poincare waves
- Three magnetostrophic waves
- Two magneto-Poincare waves and one magnetostrophic wave
- Two magnetostrophic waves and one magneto-Poincare wave

Decay Instabilities

Magneto-Poincare wave decays into two magneto-Poincare waves:

$$\frac{\partial \alpha}{\partial T_1} + a^2 \frac{\partial \alpha}{\partial X_1} + b^2 \frac{\partial \alpha}{\partial Y_1} = f^2 \gamma_0 \beta^*$$

$$\frac{\partial \beta}{\partial T_1} + a^3 \frac{\partial \beta}{\partial X_1} + b^3 \frac{\partial \beta}{\partial Y_1} = f^3 \gamma_0 \alpha^*$$

$\gamma = \gamma_0 = const$ - amplitude of pump wave

General solution: $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \beta_0 \end{pmatrix} \exp(i(\Omega T - K_x X - K_y Y))$

Growth rate: $\Gamma = \sqrt{\left| f^2 f^3 \right|} \gamma_0 > 0$

Parametric Amplifications

Initial condition $\alpha_0, \beta_0 >> \gamma$ leads to the equation

$$\frac{\partial \gamma}{\partial T_1} + a^1 \frac{\partial \gamma}{\partial X_1} + b^1 \frac{\partial \gamma}{\partial Y_1} = f^1 \alpha_0 \beta_0$$

General solution: $\gamma = \gamma_0 \exp(i(\Omega T_1 - K_x X_1 + K_y Y_1))$

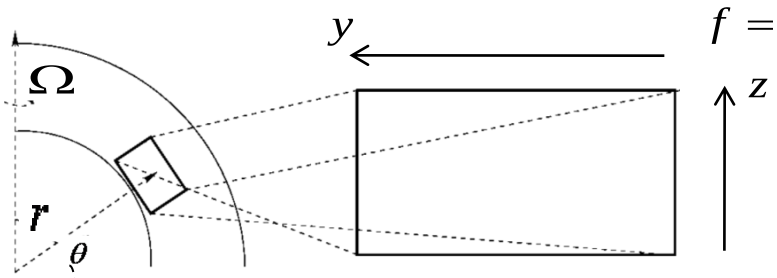
Growth rate: $\Gamma = \left| f^1 \right| \alpha_0 \beta_0 > 0$

Klimachkov D.A., Petrosyan A.S., 2017

Beta Plane Approximation

Corilois parameter :

$$f = 2\Omega \sin\theta \approx 2\Omega \sin\theta_0 + 2\Omega(\theta - \theta_0)\cos\theta_0 = f_0 + \beta y$$



$$f = f_0 + \beta y$$

Ω - angular velocity θ - latitude

Shallow water equations in beta plane approximation:

$$\frac{\partial h}{\partial t} + \frac{\partial h v_x}{\partial x} + \frac{\partial h v_y}{\partial y} = 0$$

$$\frac{\partial^2 h v_x}{\partial t \partial y} + \frac{\partial^2 (h v_x^2 - h B_x^2)}{\partial x \partial y} + \frac{\partial^2 (h v_x v_y - h B_x B_y)}{\partial y^2} + gh \frac{\partial^2 h}{\partial x \partial y} + g \frac{\partial h}{\partial x}\frac{\partial h}{\partial y} + B_0 \frac{\partial B_x}{\partial y} - f_0 \frac{\partial h v_y}{\partial y} - \beta h v_y = 0$$

$$\frac{\partial h v_y}{\partial t} + \frac{\partial (h v_x v_y - h B_x B_y)}{\partial x} + \frac{\partial (h v_y^2 - h B_y^2)}{\partial y} + gh \frac{\partial h}{\partial y} + B_0 B_y + f_0 h v_x = 0$$

$$\frac{\partial h B_x}{\partial t} + \frac{\partial (h v_y B_x - h v_x B_y)}{\partial y} - B_0 v_x = 0$$

$$\frac{\partial h B_y}{\partial t} + \frac{\partial (h v_x B_y - h v_y B_x)}{\partial y} - B_0 v_y = 0$$

Klimachkov D.A., Petrosyan A.S., 2017

Magneto-Rossby waves

Dispersion relation:

$$\omega^4 - \omega^2 \left(f_0^2 + gHk^2 + 2\left(\frac{B_0}{H}\right)^2 \right) + \omega gH\beta k_x + \left(\frac{B_0}{H}\right)^2 \left(ghk^2 + \left(\frac{B_0}{H}\right)^2 \right) = 0$$

Rossby mode ($\omega << f$):

$$\omega = -\frac{(B_0/H)^2 (ghk^2 + (B_0/H)^2)}{gh\beta k_x}$$

phase speed is westward

Zaqarashvili T.V., Oliver R. et al. 2007
Klimachkov D.A., Petrosyan A.S., 2017

Instabilities of Rossby waves

Decay Instability

$\gamma = \gamma_0 = const$ - amplitude of pump wave

$$\frac{\partial \alpha}{\partial T_1} + a^2 \frac{\partial \alpha}{\partial X_1} + b^2 \frac{\partial \alpha}{\partial Y_1} = f^2 \gamma_0 \beta^*$$
$$\frac{\partial \beta}{\partial T_1} + a^3 \frac{\partial \beta}{\partial X_1} + b^3 \frac{\partial \beta}{\partial Y_1} = f^3 \gamma_0 \alpha^*$$

General solution: $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \beta_0 \end{pmatrix} \exp(i(\Omega T - K_x X - K_y Y))$

Growth rate: $\Gamma = \sqrt{|f^2||f^3|}\gamma_0 > 0$ $\quad f^{2,3} = f^{2,3}(\mathrm{k}_1, \mathrm{k}_2, \mathrm{k}_3, f_0, \beta, g, H, B_0)$

Parametric Amplification

Initial condition $\alpha_0, \beta_0 >> \gamma$ leads to the equation

$$\frac{\partial \gamma}{\partial T_1} + a^1 \frac{\partial \gamma}{\partial X_1} + b^2 \frac{\partial \gamma}{\partial Y_1} = f^3 \alpha_0 \beta_0$$

General solution: $\gamma = \gamma_0 \exp(i(\Omega T_1 - K_x X_1 + K_y Y_1))$

Growth rate: $\Gamma = |f^1| \alpha_0 \beta_0 > 0$ $\quad f^1 = f^1(\mathrm{k}_1, \mathrm{k}_2, \mathrm{k}_3, f_0, \beta, g, H, B_0)$

Klimachkov D.A., Petrosyan A.S., 2017

Damping Terms and Threshold Amplitudes

Decay Instability

$$\frac{\partial \alpha}{\partial T_1} + a^2 \frac{\partial \alpha}{\partial X_1} + b^2 \frac{\partial \alpha}{\partial Y_1} + \eta_2 \alpha = f^2 \gamma_0 \beta^*$$

$$\frac{\partial \beta}{\partial T_1} + a^3 \frac{\partial \beta}{\partial X_1} + b^3 \frac{\partial \beta}{\partial Y_1} + \eta_3 \beta = f^3 \gamma_0 \alpha^*$$

$\eta_2 \alpha, \eta_3 \beta$ - damping terms

Critical amplitude: $$\gamma_0^{cr} = \frac{\sqrt{\eta_2 \eta_3}}{\sqrt{\left| f^2 \right| \left| f^3 \right|}}$$

Growth rate: $$\Gamma = \sqrt{\left| f^2 \right| \left| f^3 \right|} \gamma_0^{cr} > 0$$

Parametric Amplification

$$\frac{\partial \gamma}{\partial T_1} + a^1 \frac{\partial \gamma}{\partial X_1} + b^1 \frac{\partial \gamma}{\partial Y_1} + \eta_1 \gamma = f^1 \alpha_0 \beta_0$$

$\eta_1 \gamma$ - damping term

Critical amplitude: $$(\alpha_0 \beta_0)^{cr} = \frac{\eta_1}{f^1}$$

Growth rate: $$\Gamma = \left| f^1 \right| (\alpha_0 \beta_0)^{cr} > 0$$

2D β-plane magnetohydrodynamic equations

$$\omega_t = J(\psi, \omega) + \beta\psi_x + J(A, \Delta A) - B_0 \Delta A_x + \nu\Delta\omega + F$$

$$A_t = J(\psi, A) + B_0\psi_x + \mu\Delta A$$

$$\omega = -\Delta\psi$$

ω - vorticity

ψ - stream function

A - magnetic potential

β - Rossby parameter

B_0 - toroidal component of magnetic field

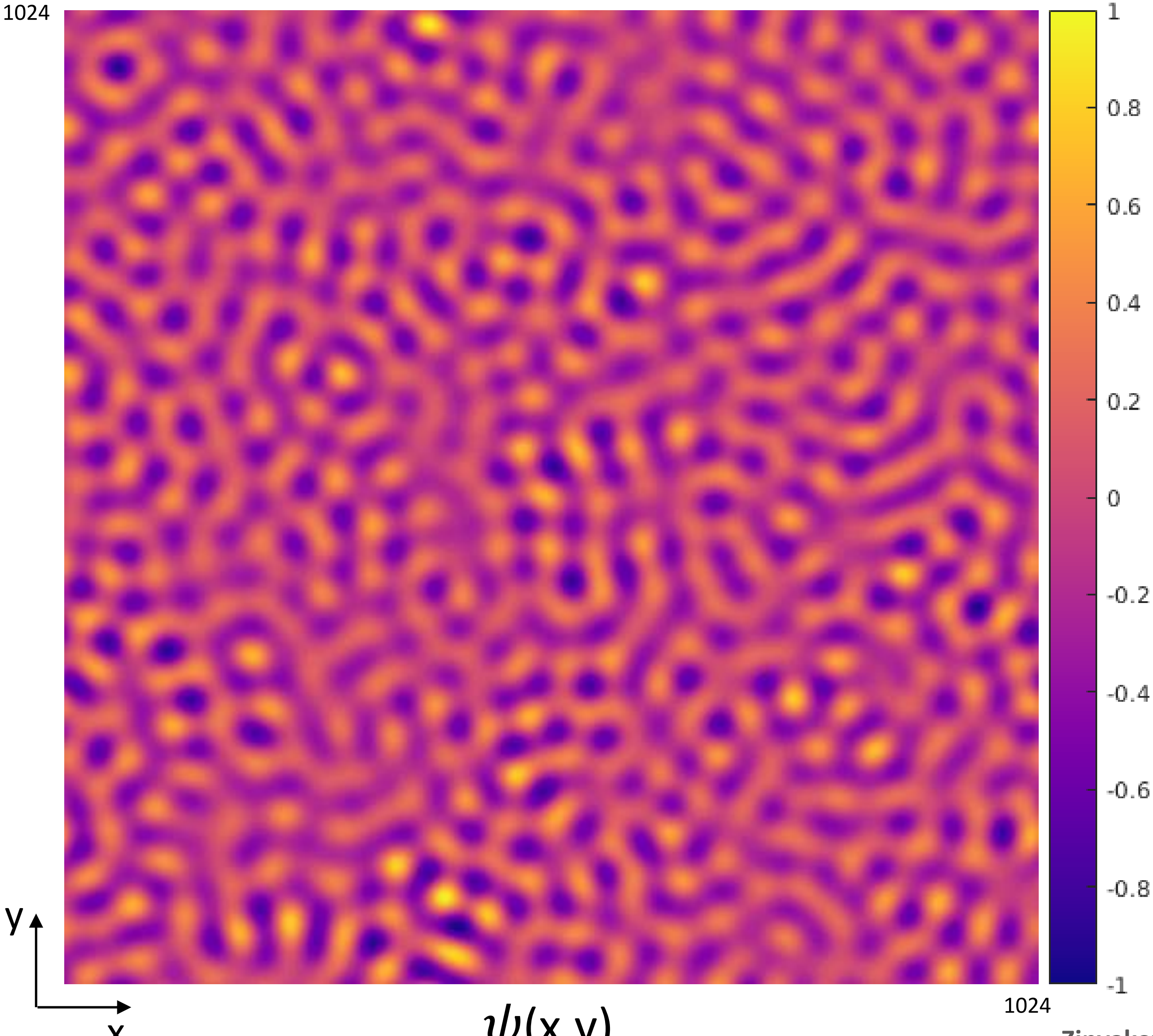
$J(a, b) = a_x b_y - a_y b_x$ - Jacobian

ν - kinematic viscosity

μ - magnetic diffusivity

F - forcing

Zonal flows in decaying 2D β-plane hydrodynamic turbulence



ψ(x,y)

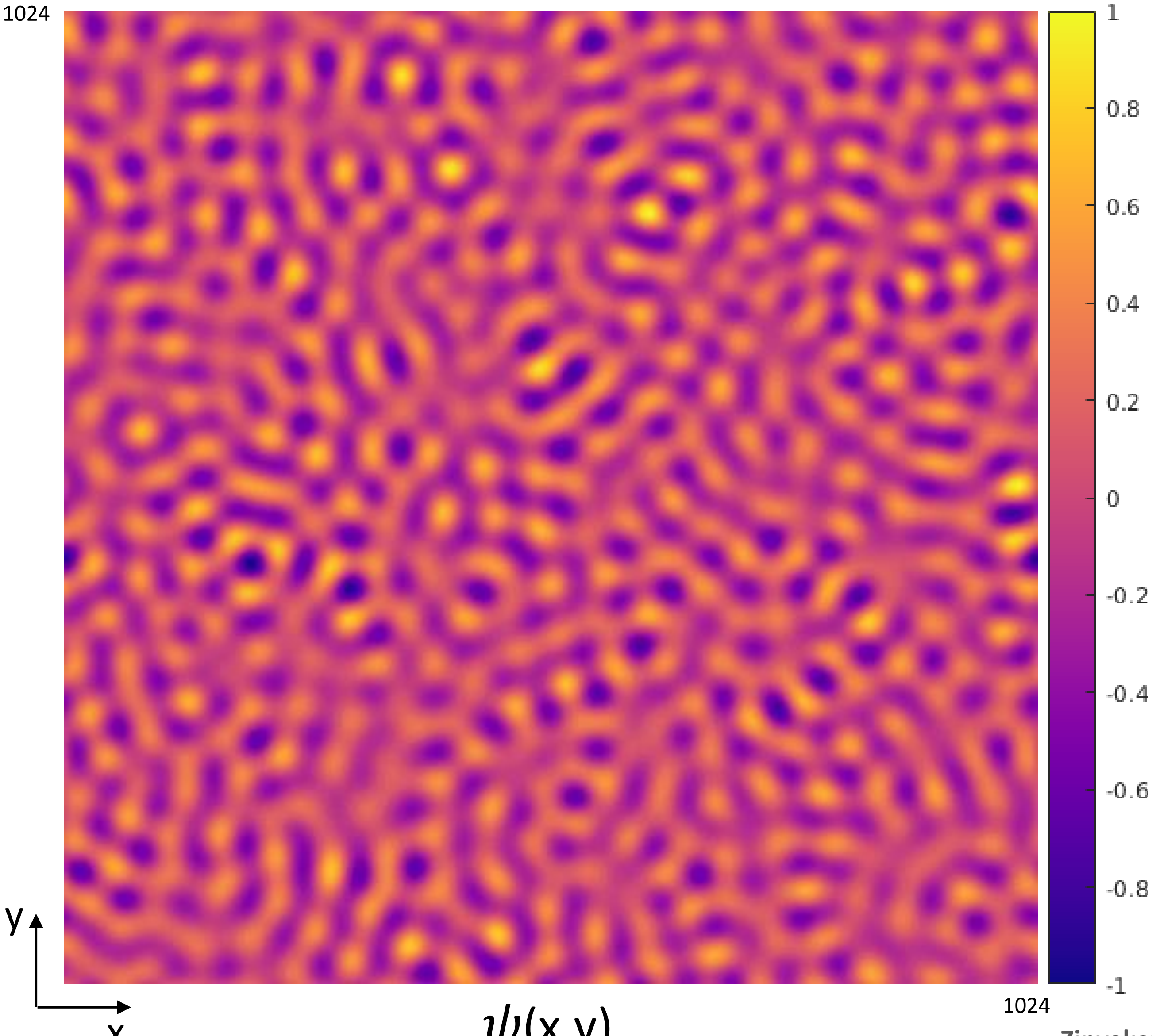
$E^0_{mag} = 0$

$\beta = 10$

- Zonal flows are formed

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Zonal flows in decaying β-plane 2D magnetohydrodynamic turbulence



ψ(x,y)

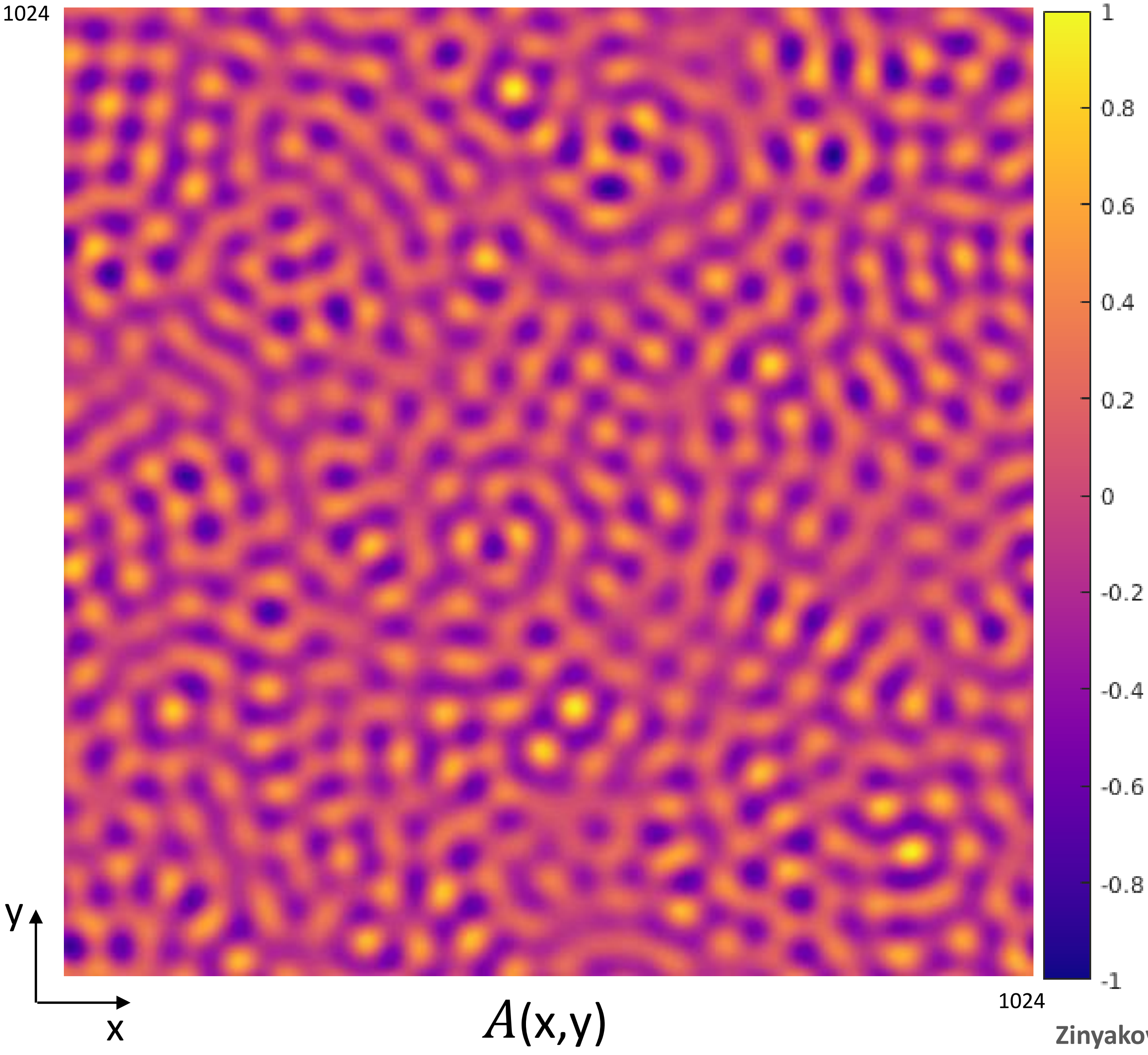
$$E^0_{kin} = E^0_{mag}$$

$$\beta = 10$$

- Time dynamics of zonal flows

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Magnetic field in decaying 2D β-plane magnetohydrodynamic turbulence



A(x,y)

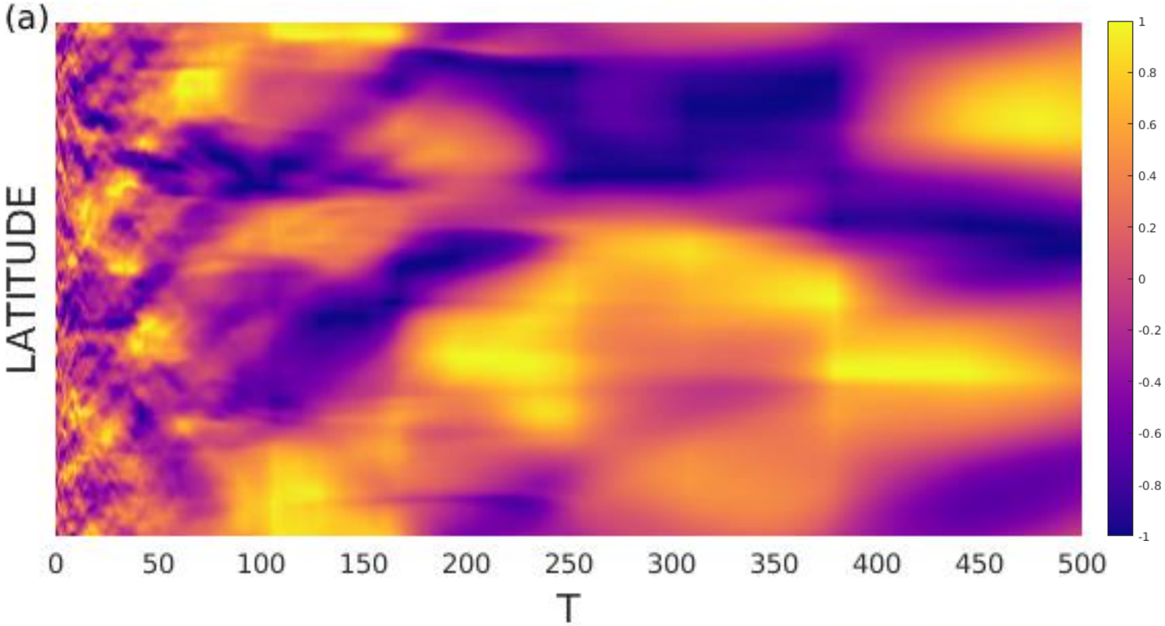
$$E^0_{kin} = E^0_{mag}$$

$$\beta = 10$$

- Magnetic field lines are aligned perpendicular to the zonal flows because of the effect of magnetic freezing

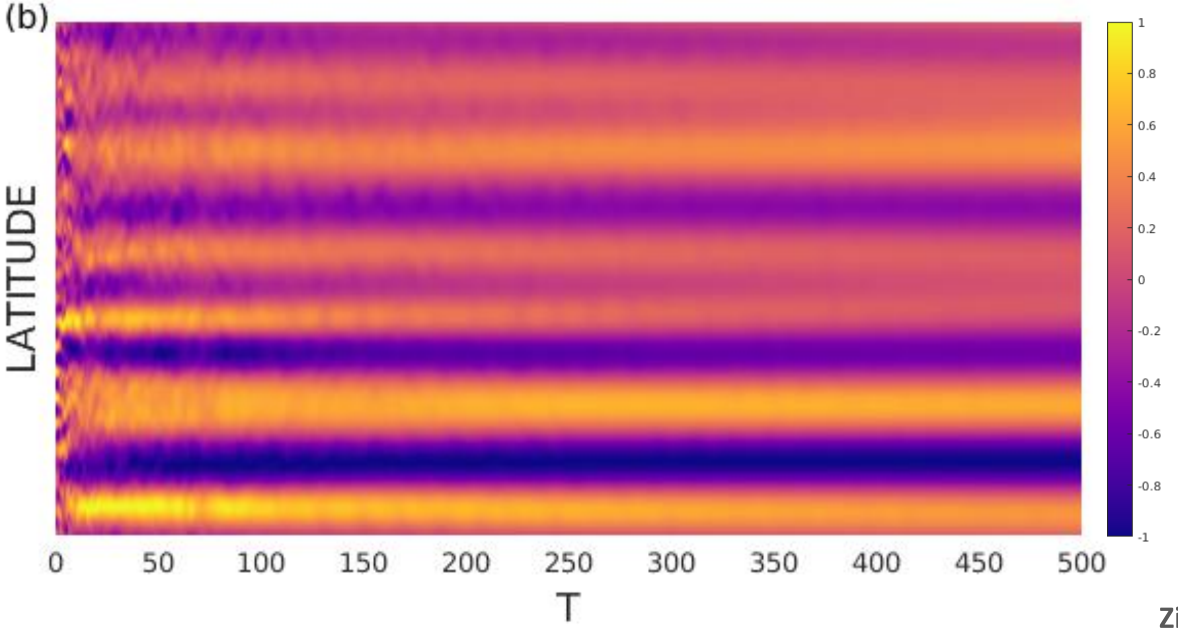
Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Mean zonal velocity $\bar{u}(y)$



- Decaying 2D β-plane magnetohydrodynamic turbulence

$$E^0_{kin} = E^0_{mag}$$



- Decaying 2D β-plane turbulence

$$E^0_{mag} = 0$$

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Anisotropic scale in magnetohydrodynamic turbulence

The eddy turnover time:

$$\tau_k = \frac{2\pi}{U\,k}$$

$$\frac{1}{\tau_k} = \omega_{turb} = \omega_{Rossby} \quad \Longrightarrow$$

The dispersion relation for Rossby waves:

$$\omega = -\frac{\beta k_x}{k^2}$$

$$k_\beta = \sqrt{\frac{\beta}{2U}}$$ - Rhines Scale

The magnetic eddy turnover time:

$$\tau_k^M = \frac{l_k U}{B^2} = \frac{2\pi\, U}{B^2\, k}$$

$$\frac{1}{\tau_k^M} = \omega_{mag} = \omega_{Rossby} \quad \Longrightarrow$$

The dispersion relation for Rossby waves:

$$\omega = -\frac{\beta k_x}{k^2}$$

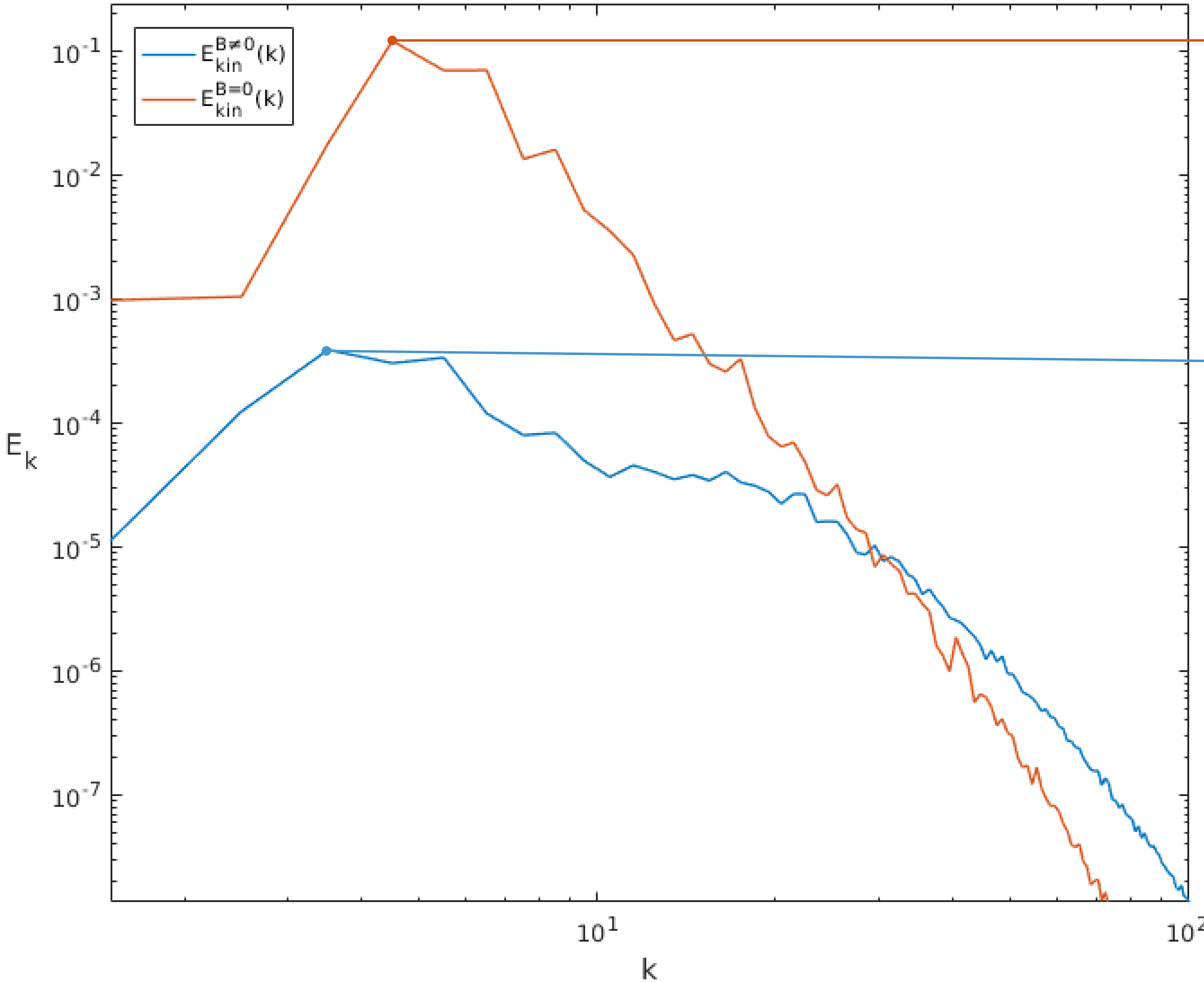
$$k_\beta^M = \sqrt{\frac{\beta U}{2B^2}}$$ - Anisotropic scale in magnetohydrodynamic turbulence

U - root-mean-square velocity

B - root-mean-square magnetic field

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Spectrum of decaying 2D magnetohydrodynamic turbulence



$k^{max} = 4.5$

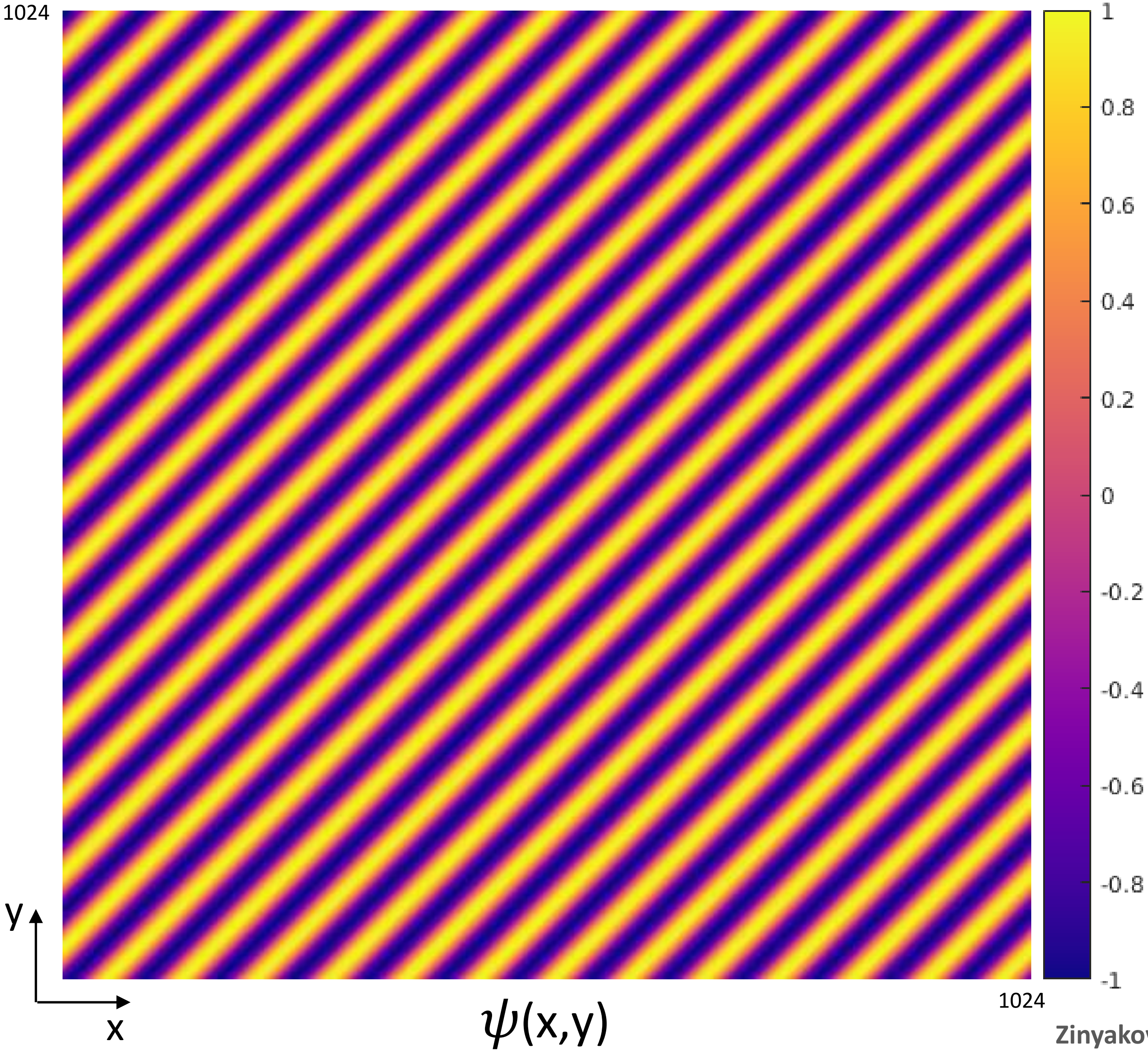
$k_\beta = \sqrt{\frac{\beta}{2U}} \approx 4.7$

$k^{max} = 3.5$

$k_\beta^M = \sqrt{\frac{\beta U}{2B^2}} \approx 3.7$

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Zonal flows in 2D β-plane hydrodynamicturbulence



ψ(x,y)

$B_0 = 0$

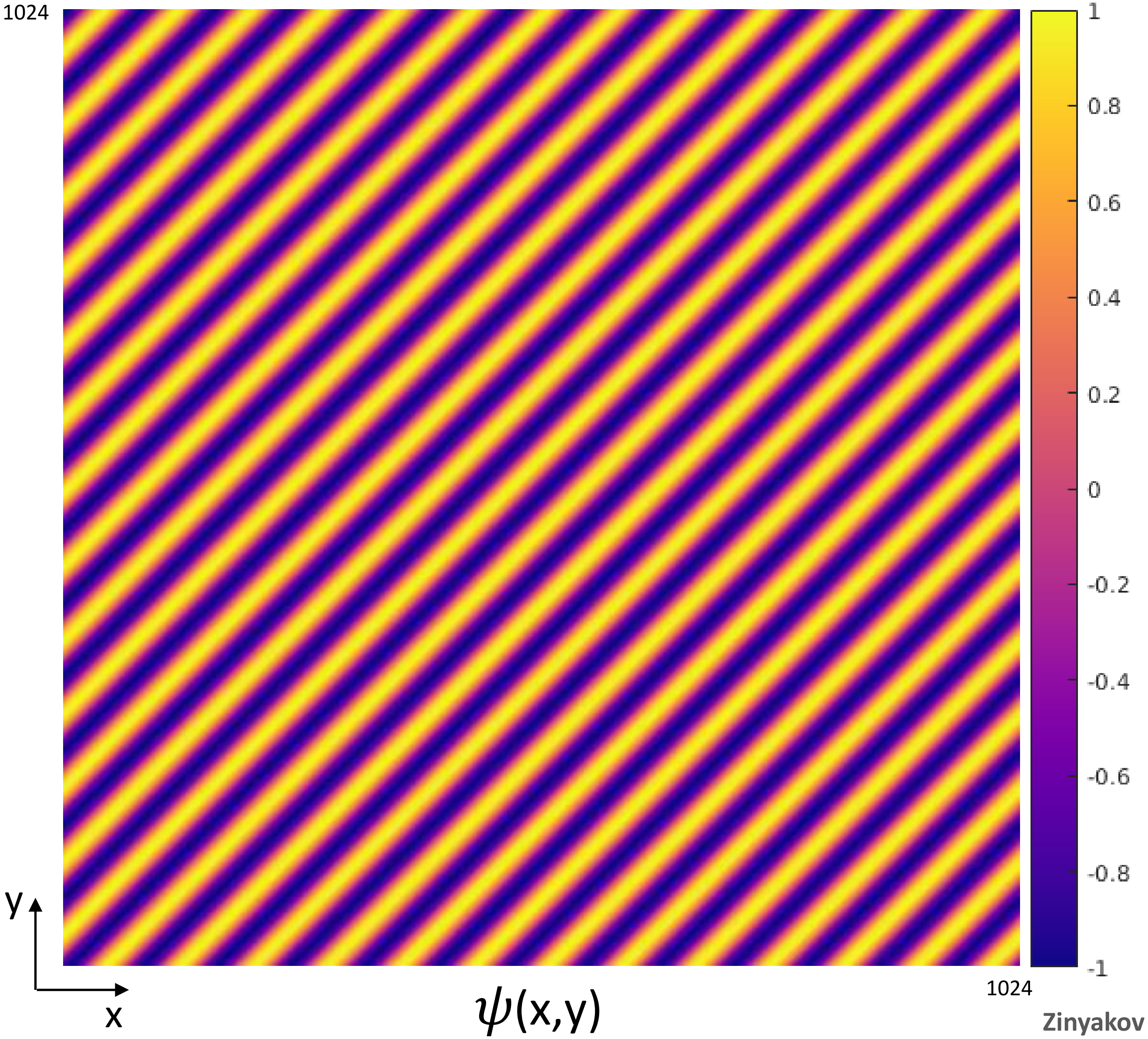
β =5

Forcing at wavenumbers
$14 \leq |k| \leq 20$

- Zonal flows are formed

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Zonal flows in 2D β-plane magnetohydrodynamic turbulence in toroidal magnetic field



$B_0 = 10^{-3}$

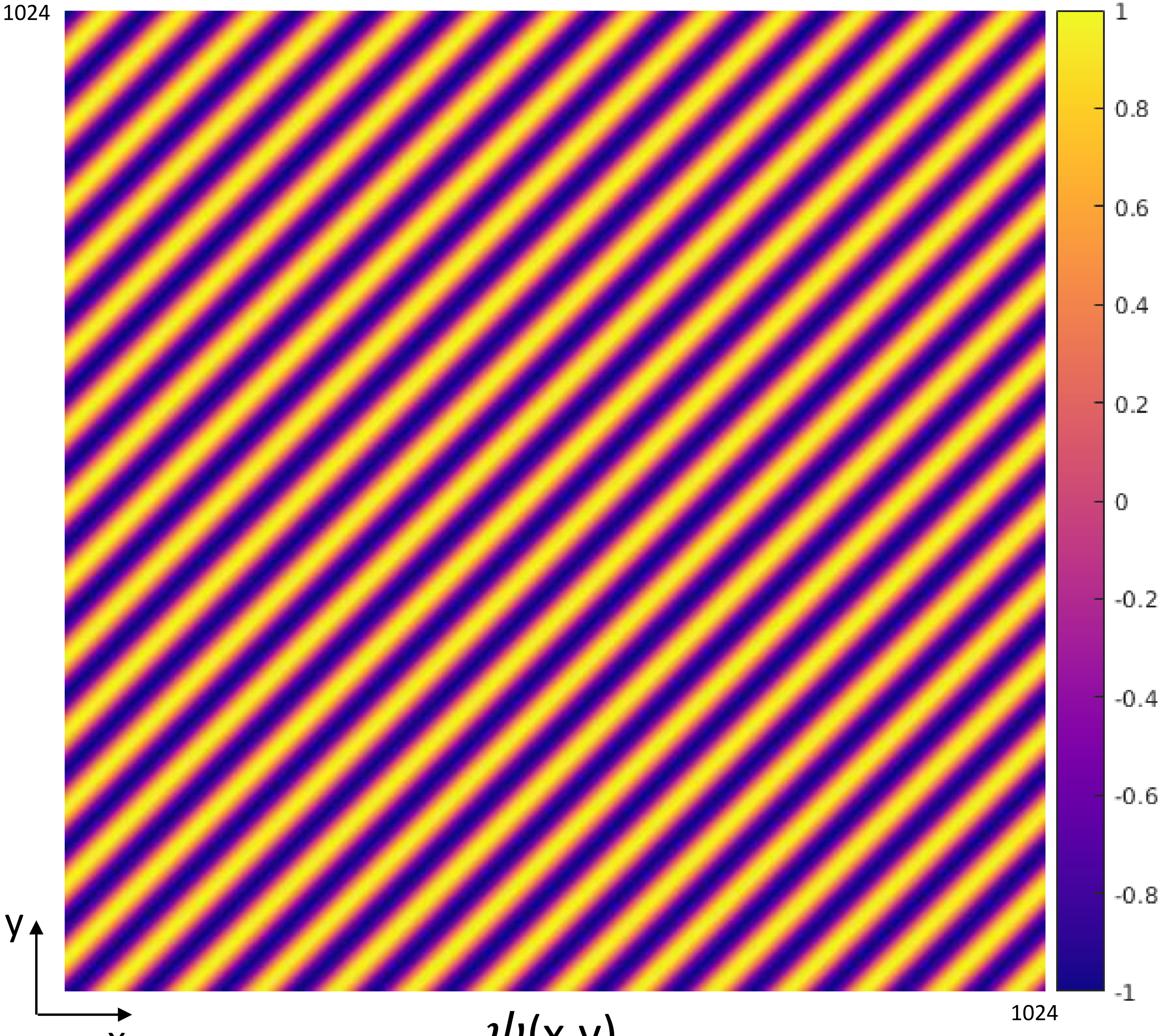
β =5

Forcing at wavenumbers
$14 \leq |k| \leq 20$

- Zonal flows are formed

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

2D β-plane magnetohydrodynamic turbulence in toroidal magnetic field



ψ(x,y)

$$B_0 = 10^{-2}$$

$$\beta = 5$$

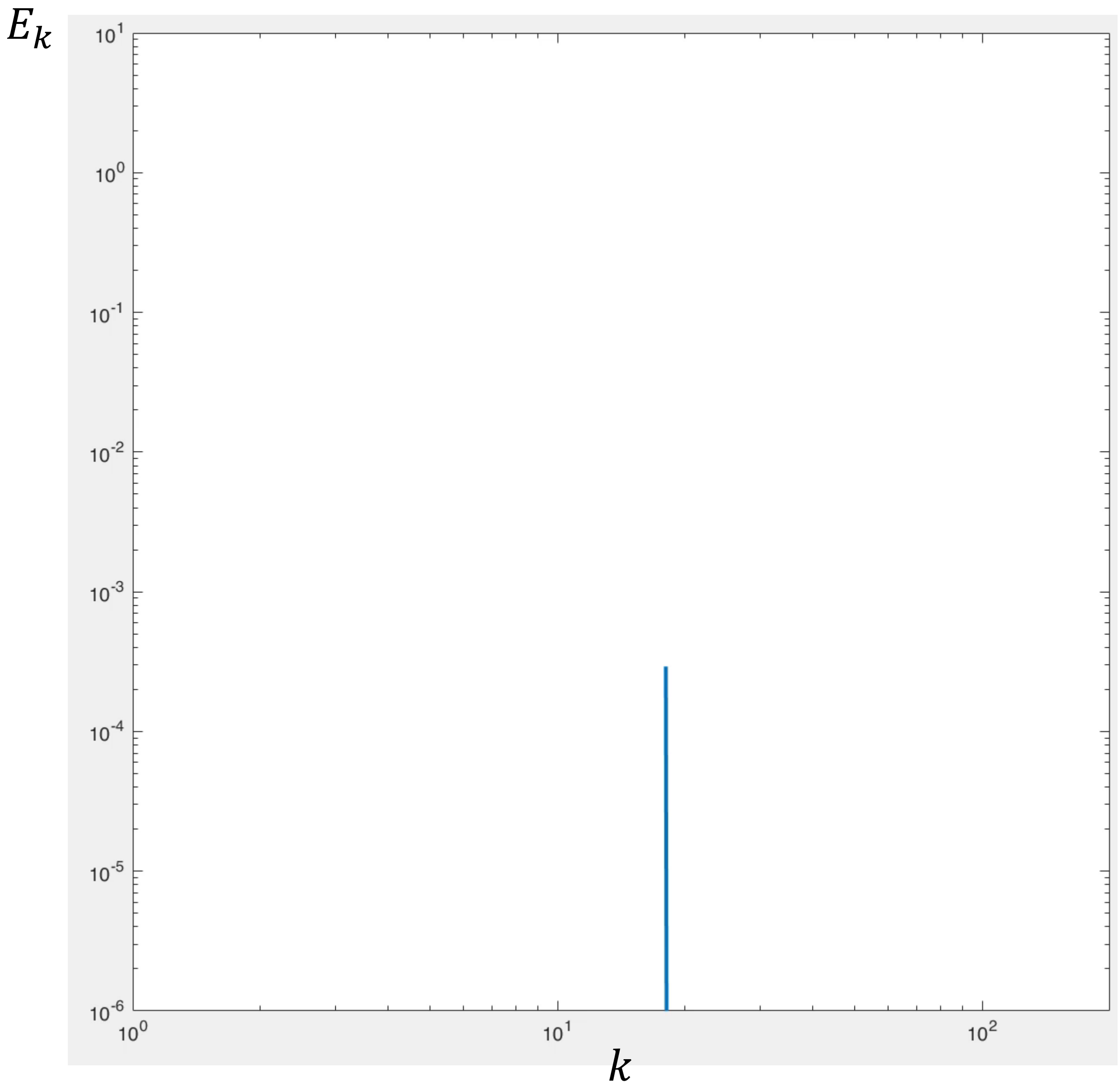
Forcing at wavenumbers

$$14 \leq |k| \leq 20$$

- Zonal flows aren't formed
- The inverse cascade is halted by the presence of the magnetic field

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Spectrum of 2D β-plane magnetohydrodynamic turbulence in toroidal magnetic field



$\beta = 5$ $\quad B_0 = 10^{-2}$

Forcing at wavenumbers

$14 \leq |k| \leq 20$

- The inverse cascade is halted by the presence of the magnetic field
- There isn't transfer of energy from forced scales to large scales

Zinyakov T.A. and Petrosyan A.S., 2018 (submitted)

Conclusions

- Rotating MHD shallow water equations in external magnetic field, beta plane approximation
- Linear waves, nonlinear amplitudes equations in Shallow Water Magnetohydrodynamics
- Parametric instabilities in Shallow Water Magnetohydrodynamics
- Beta plane magnetohydrodynamic turbulence, zonal flows
- Anisotropic scale in magnetohydrodynamic turbulence

THANK YOU FOR YOUR ATTENTION

Numerical Simulation

- Square box of size 2 π * 2 π with periodic boundary conditions
- 1024*1024 collocation points in space
- Pseudospectral method
- Dealiazing according to the 2/3 rule

- CUDA Parallel Technology with using CUDA C++
- Nvidia Tesla k40