О взаимодействии мелкомасштабной турбулентности с течениями и внутренними волнами в стратифицированной среде

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Краткая история

- В стратифицированных течениях (океане, атмосфере) турбулентность поддержиается сдвигом скорости и подавляется устойчивой стратификацией плотности.
- Классический подход [1,2] здесь основан на усредненных уравнениях Рейнольдса, дополненных гипотезами замыкания колмогоровского типа,
- Число Ричардсона Ri: в классической теории развитая турбулентность может поддерживаться и усиливаться при условии $Ri \leq 1$, и в ряде случаев результаты теории неплохо согласуются с наблюдениями. Однако во многих других случаях турбулентность существует при гораздо больших значениях Ri.
- Соответствующая модификация теории была предложена в работе [3], где использовалось кинетическое уравнение для функции распределения флуктуаций скорости и плотности. В результате ограничение на величину Ri снимается вообще: энергия турбулентности может оставаться конечной при любом конечном Ri.

[1] Монин, А.М., Яглом, А. М. Статистическая гидромеханика, т.1, М., Наука, 1965.

[2] Rodi, W. In: Prediction Methods for Turbulent Flows, ed. Kollman, W., Hemisphere, New York, pp. 259–350. 1980.

[3] Островский, Л. А., Троицкая Ю. И. Изв. АН СССР, Физ. Атм. и Океана, т. 23. № 10, сс. 1031-1040, 1987.

General equations

(Ostrovsky and Troitskaya, FAO, 23 (10)1987)

$$\frac{\partial\langle u_i\rangle}{\partial t}+\langle u_j\rangle\frac{\partial\langle u_i\rangle}{\partial x_j}+\frac{1}{\rho_0}\frac{\partial\langle p\rangle}{\partial x_i}+g_i\frac{\langle\rho\rangle-\rho_0}{\rho_0}=\frac{\partial}{\partial x_j}\left(L\sqrt{K}\left(\frac{\partial\langle u_i\rangle}{\partial x_j}+\frac{\partial\langle u_j\rangle}{\partial x_i}\right)\right),$$

$$\frac{\partial\langle\rho\rangle}{\partial t}+\langle u_i\rangle\frac{\partial\langle\rho\rangle}{\partial x_i}=2\frac{\partial}{\partial x_i}L\sqrt{K}\left(\frac{\partial\langle\rho\rangle}{\partial x_i}+\frac{3}{2k\rho_0}(g_i\langle\rho'^2\rangle-g\beta_i)\right),$$

$$\frac{\partial K}{\partial t}+\langle u_i\rangle\frac{\partial K}{\partial x_i}-L\sqrt{K}\left(\frac{\partial\langle u_i\rangle}{\partial x_j}+\frac{\partial\langle u_j\rangle}{\partial x_i}\right)^2-\frac{g}{\rho_0}L\sqrt{K}\times$$

$$\times\left(\frac{\partial\langle\rho\rangle}{\partial z}+\frac{3g}{2k\rho_0}(\langle\rho'^2\rangle-\beta_z)\right)+\frac{CK^{3/2}}{L}=\frac{5}{3}\frac{\partial}{\partial x_i}\left(L\sqrt{K}\frac{\partial K}{\partial x_i}\right),$$

$$\frac{\partial\langle\rho'^2\rangle}{\partial t}+\langle u_i\rangle\frac{\partial\langle\rho'^2\rangle}{\partial x_i}-2\frac{\partial\langle\rho\rangle}{\partial x_i}L\sqrt{K}\left(\frac{\partial\langle\rho\rangle}{\partial x_i}+\frac{3}{2K\rho_0}(g_i\langle\rho'^2\rangle-g\beta_i)\right)+$$

$$+\frac{DK^{1/2}}{L}\langle\rho'^2\rangle=\frac{\partial}{\partial x_i}L\sqrt{K}\frac{\partial\langle\rho'^2\rangle}{\partial x_i}.$$

$\langle\boldsymbol{u}\rangle$ is the ensemble-average velocity, $\langle\rho\rangle$ is average density, ρ **’** – its fluctuations, $\boldsymbol{g}$ – gravity, L is outer scale of turbulence, C и D – empirical constants, $\boldsymbol{K=3\,\langle u'^2\rangle/2}$ – kinetic energy of turbulence. The last eq. defines the potential energy of turbulence:

$$\boldsymbol{P=\frac{\langle\rho'^2\rangle g^2}{2N^2\rho_0^2}}$$

Local interactions at given N^2 and dV/dz

Gladskikh et al, J. Marine Sci. Eng. , 11, 136, 2023

If the velocity shear and average density profile are given and diffusion is neglected, we have local equations for K and P (horizontal velocity, vertical gradients):

$$\frac{dK}{dt} = L\sqrt{K}\left(\frac{dV_0}{dz}\right)^2 - N^2 L\sqrt{K}\left(1-\frac{3P}{K}(1-G)\right) - \frac{CK^{3/2}}{L}$$

$$\frac{dP}{dt} = N^2 L\sqrt{K}\left(1-\frac{3P}{K}(1-G)\right) - \frac{DPk^{1/2}}{L}$$

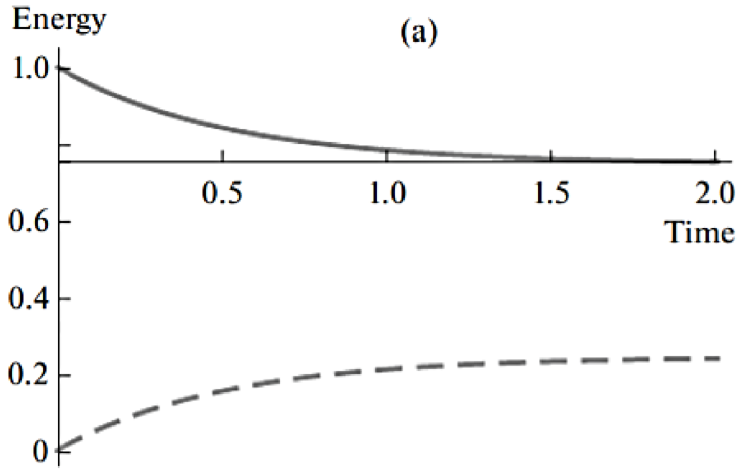
$$N = \sqrt{-(g/\langle\rho\rangle)d\langle\rho\rangle/dz}$$

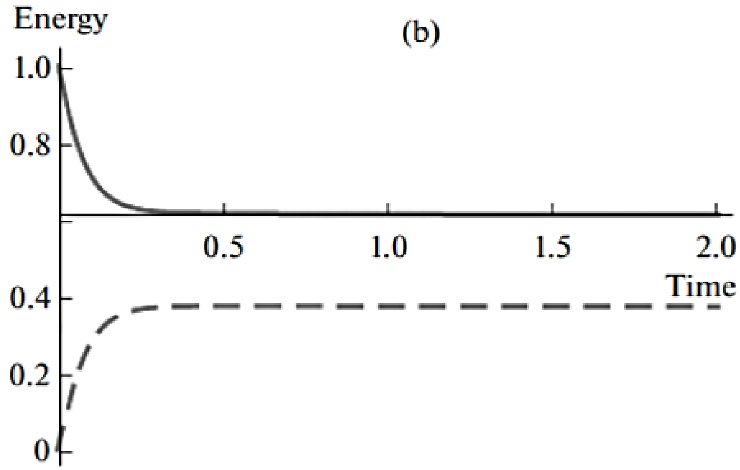
Here N is Brunt-Vaisala (buoyancy) frequency, L is the turbulence length scale, $G = \beta_z / \langle\rho'^2\rangle$ is the anisotropy parameter (taken 0.5 here), C and D are the empirical constants; here $D = C$ is taken.

Richardson number:

***Ri* = *N*^2/*V*$_z$^2**, where $V_z = dV/dz$ –average velocity shear

Transient processes





Dimensionless kinetic energy $K = \frac{K}{K_x}, K_x = \frac{V_{0z}^2 L^2}{2C}$ (solid line) and potential energy $p = \frac{P}{P_x}, P_x = \frac{V_{0z}^2 L^2}{C}$ (dashed line) of turbulence vs. dimensionless time $t = \frac{t}{t_x}, t_x = \frac{1}{V_{0z} C}$ for *Ri* = 0.5 (a) and *Ri* = 5 (b)

Stationary solutions

$$\boxed{\begin{aligned} K_{st} &= \frac{V_{0z}^2}{2C} f(Ri), \\ P_{st} &= \frac{V_{0z}^2}{C}\left(1 - \frac{f(Ri)}{2}\right) \end{aligned}}$$

$$f(Ri) = 1 - (4-3G)Ri + \left[1 + Ri^2(4-3G)^2 + (4-6G)Ri\right]^{\frac{1}{2}}$$

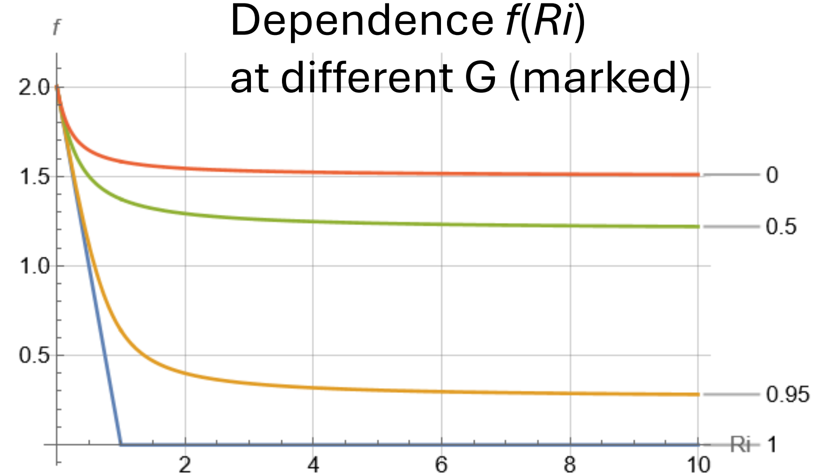
The $K-\varepsilon$ **case** corresponds to G =1 and the equations are separated. Stationary limits are:

$$K_{st} = \frac{V_{0z}^2}{C}(1 - Ri),$$

$$P_{st} = \frac{V_{0z}^2}{C} Ri, \quad Ri \le 1$$

In this case the threshold is *Ri* = 1

General case:
Dependence *f*(*Ri*)
at different G (marked)



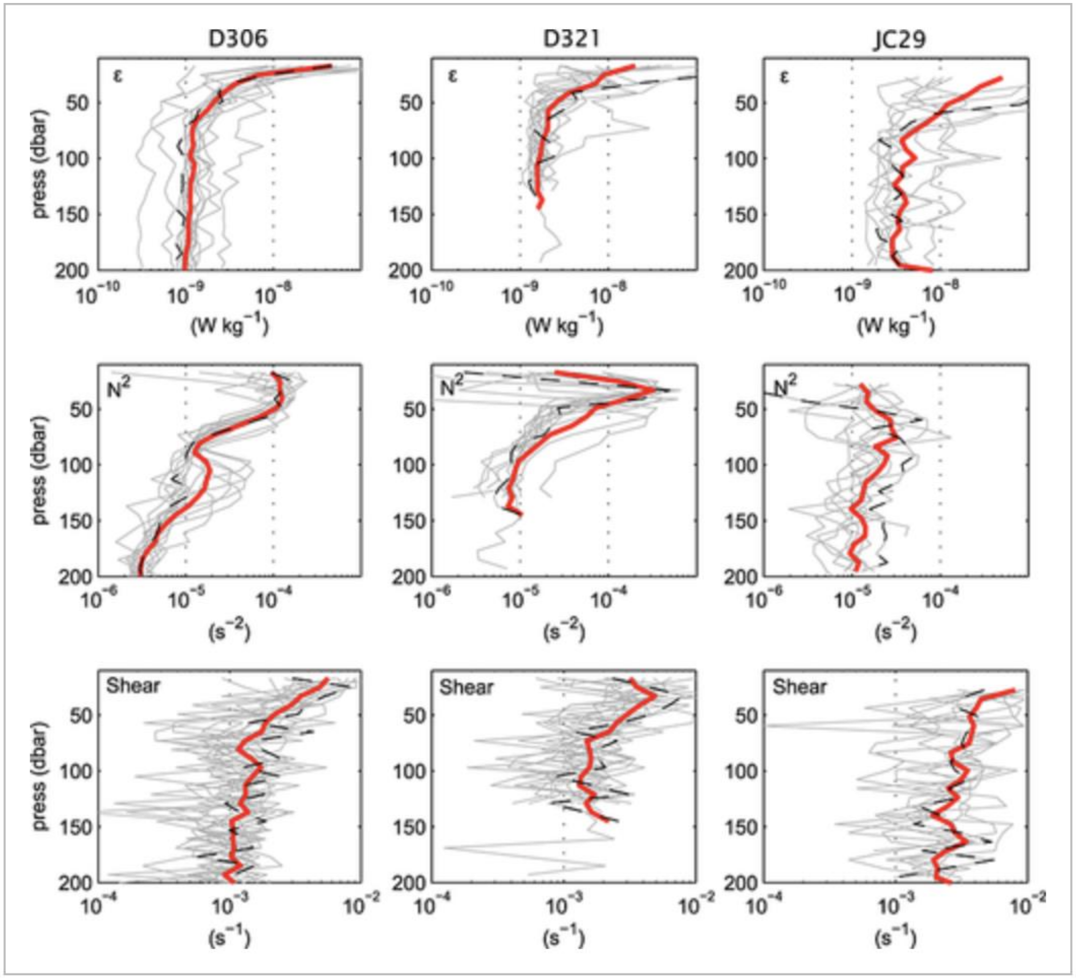
In the general case always f < 2 for all Ri > 1 so that *K*,*P* > 0

Oceanic experiments

Forryan, A., et al., J. Geophys. Res. Oceans, 118, 1405– 1419, 2013

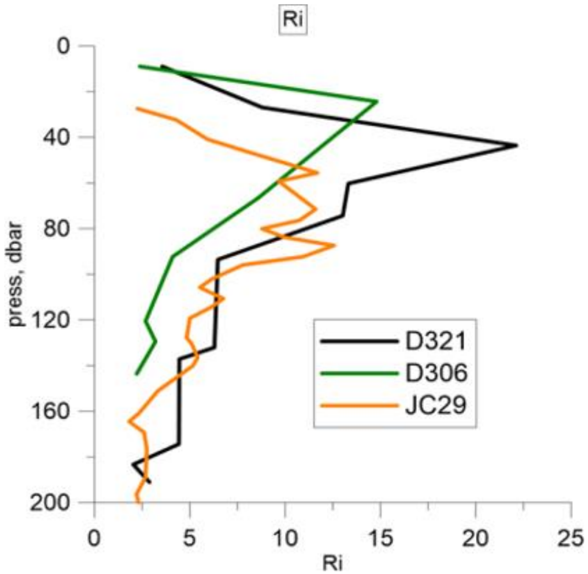
Th
UK RSS Discovery:
- D306 (Абиссальная равнина, июнь – июль 2006 г.)
- D321 (Исландский бассейн, июль - август 2007 г.)

UK RSS James:
- JC29 (плато Кергелен, ноябрь - декабрь 2008 г.)



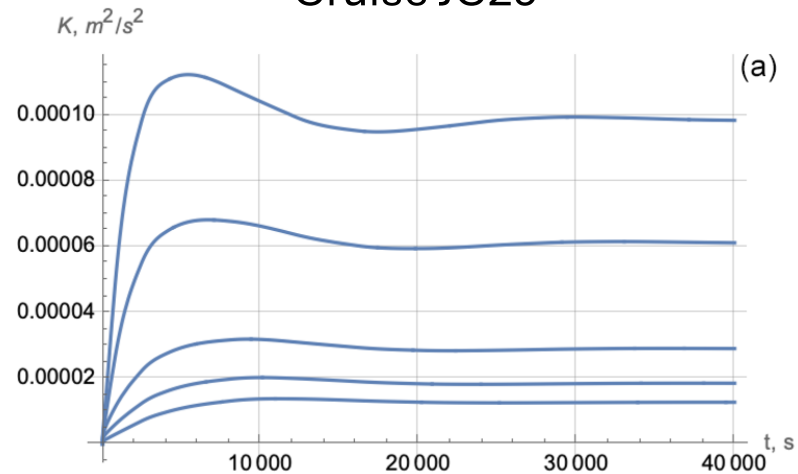
Theory vs data

Ostrovsky et al., Nonlin. Proc. Geophys. 31, 219– 227, 2024

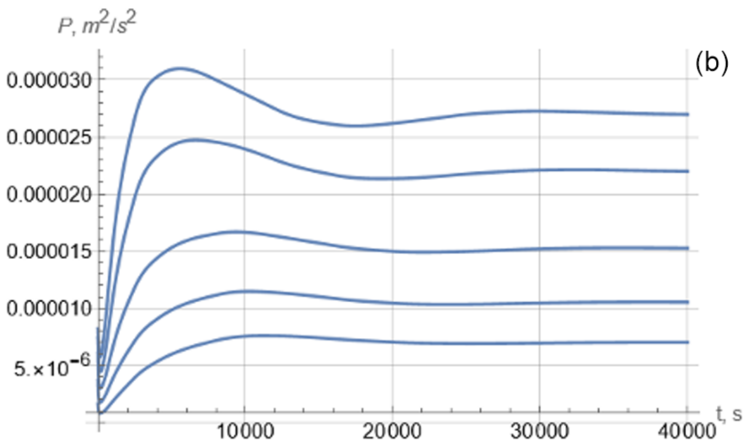


Richardson number

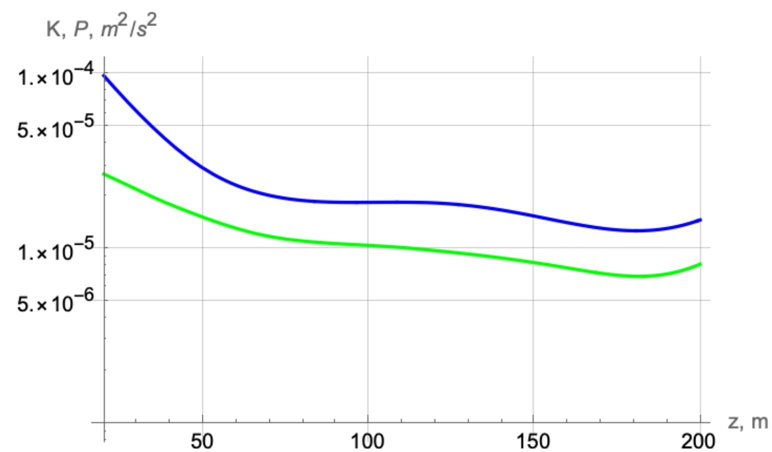
Cruise JC29



Transient process: K(t) for different depths z= 20,30,50,100,180 m



Transient process: $P(t)$ for different z

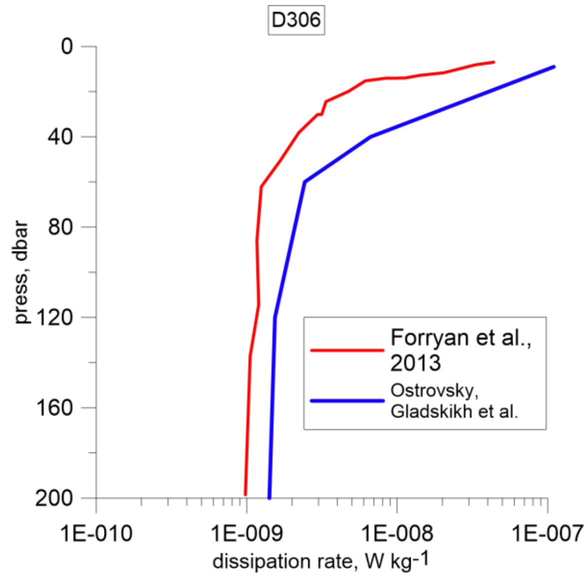


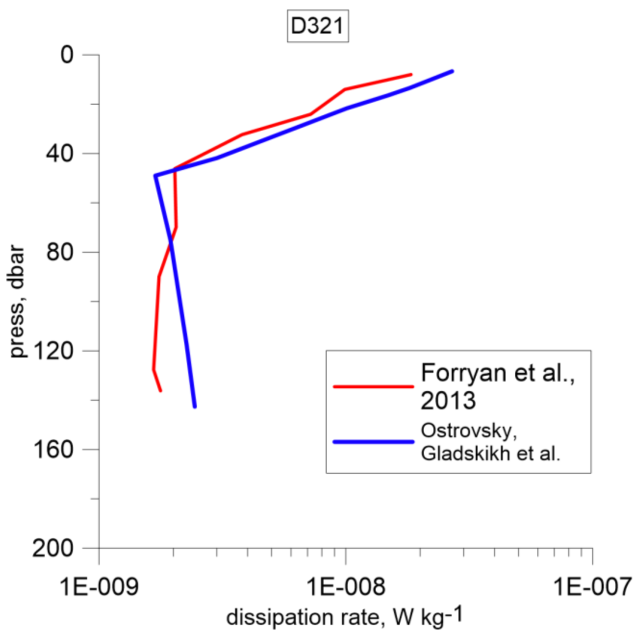
Stationary values: $K(z)$ (blue) and $P(z)$ (green)

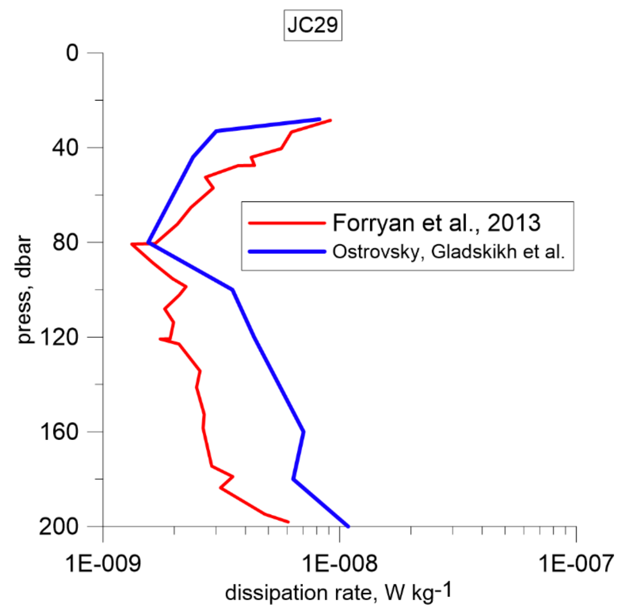
Dissipation rate of turbulent kinetic energy

$$\boxed{\varepsilon = \frac{CK^{3/2}}{L}}$$

Red-from Forryan et al, 2013
Blue –our calculations

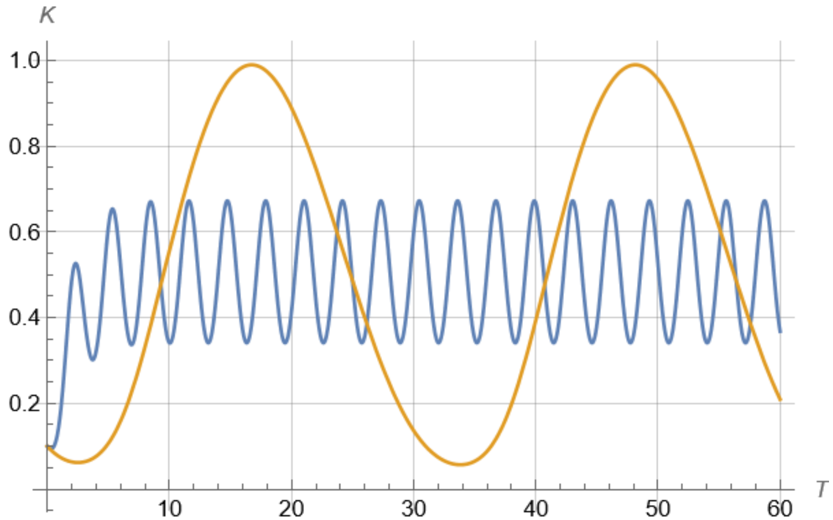






Effect of internal waves

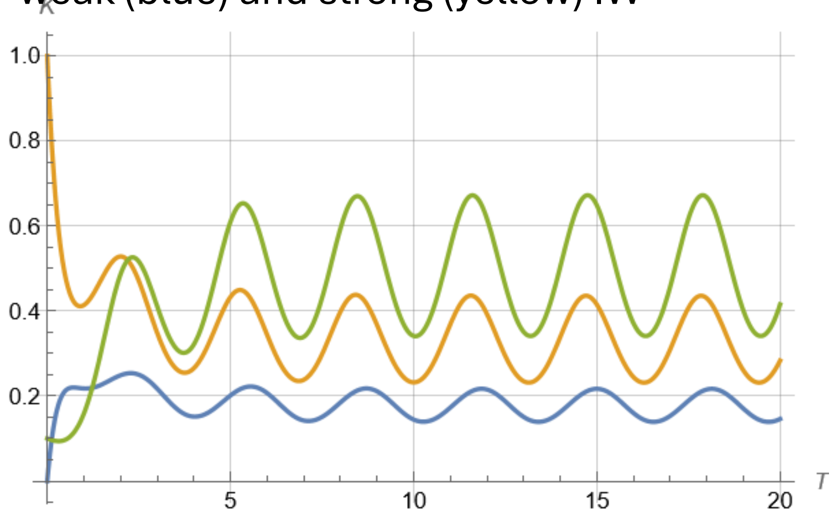
Here $V = A(z)\sin(\omega t - kx)$



Evolution of turb kinetic energy (TKE) affected by weak (blue) and strong (yellow) IW

Interaction of IW and small-scale turbulence is twofold: the IW damping and the action of IW on turbulence. Here we consider only the latter.

Classical closures, after Ivanov, A. et al. Dyn. Atm. Oceans, 7, 221-232, 1983



Full model, same parameters
Weak IW (yellow), strong IW (green)
Potential energy (blue)

Experiment with internal waves

Matusov et al. Sov. Phys. Doklady, 307 n.4, 979, 1989

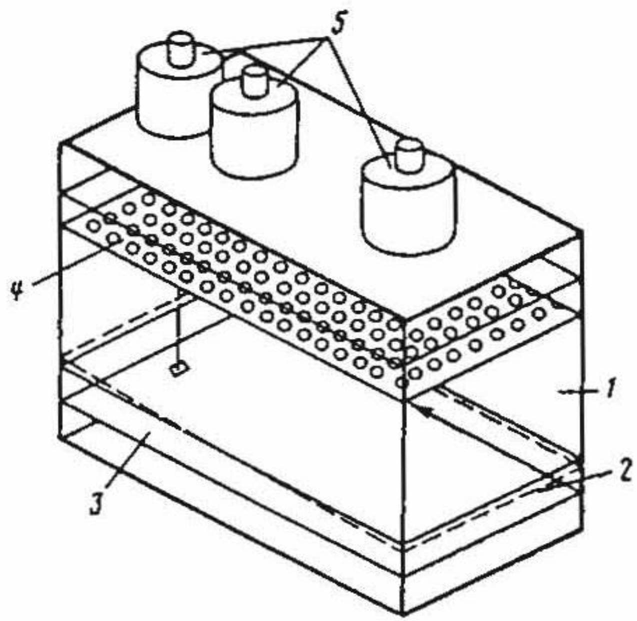


Fig. 1

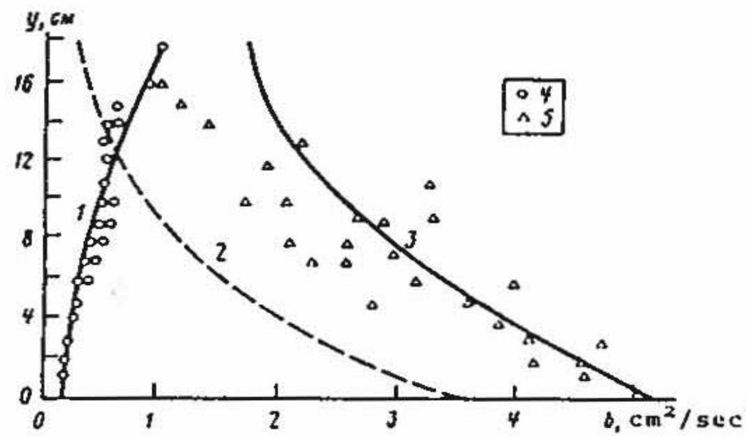


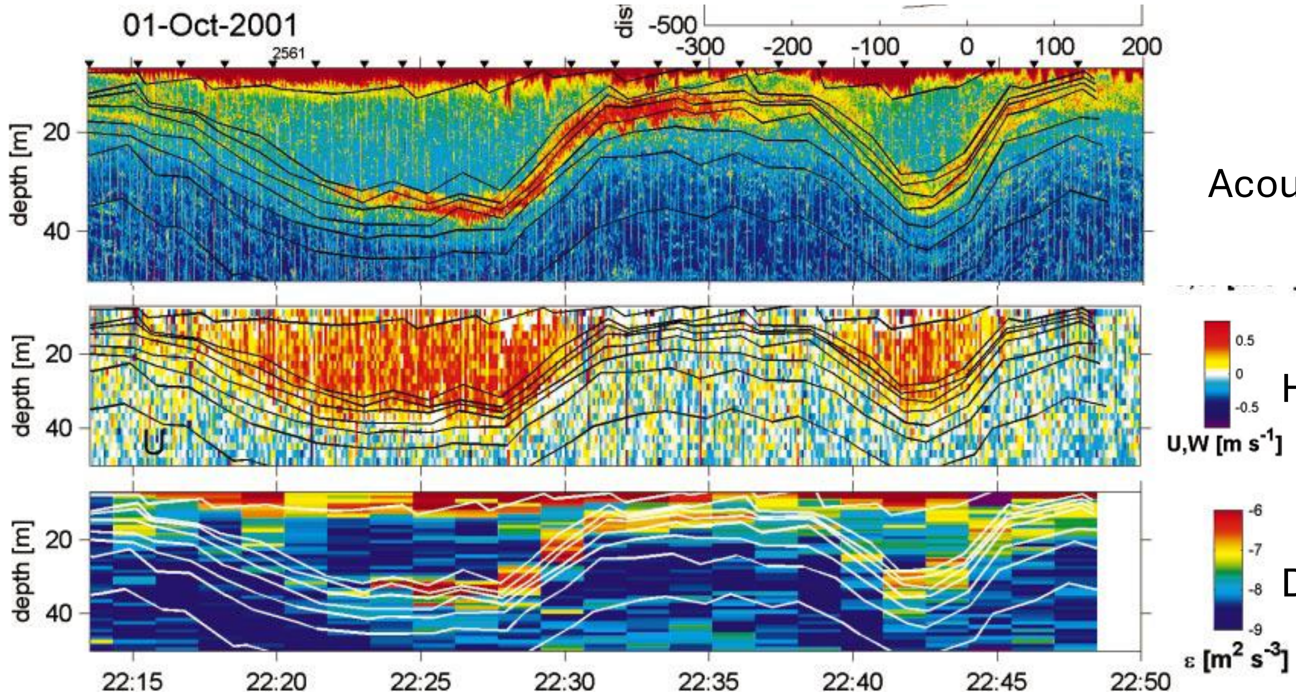
Fig. 2

Fig. 1. Scheme of the experiment. 1 – Water, 2- Freon, 3-Internal wave generator, 4-Perforatef grid, 5 – Wave generator and grid-drive mechanisms.

Fig. 2. Vertical distribution of turbulent energy. 1-3 Theory, 4, 5 – experiment. 1, 4 –Turb. without IW, 2, 3, 5 – Turb. in the presence of IW.

Experiment with internal waves

Moum, J.N. et al. J. Phys. Oceanogr. 33,2093-2112, 2003
Oregon shelf, 2001

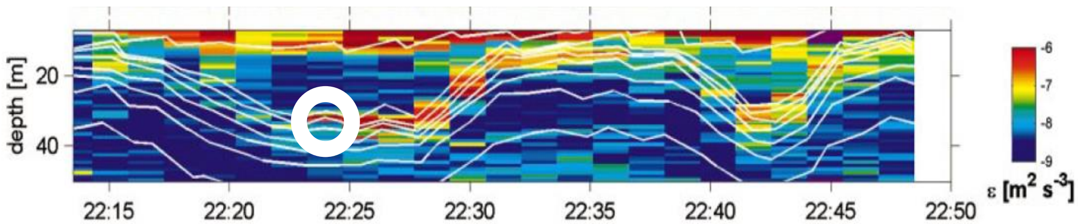


Acoustic scattering level

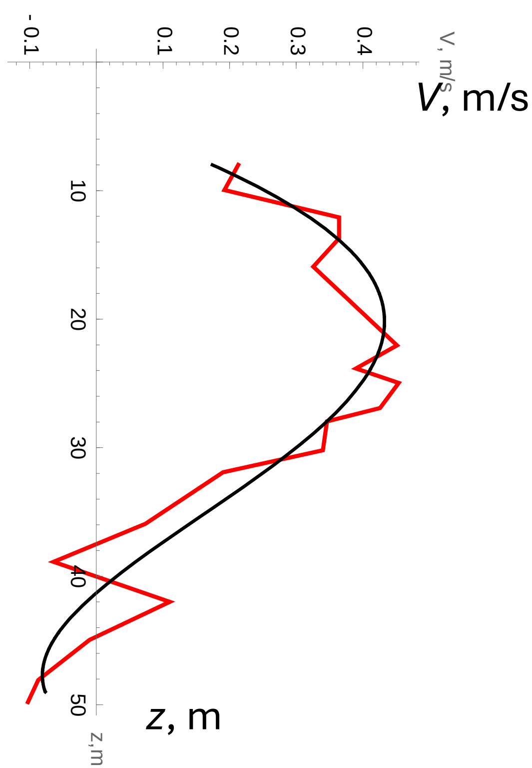
Horizontal velocity

Dissipation rate

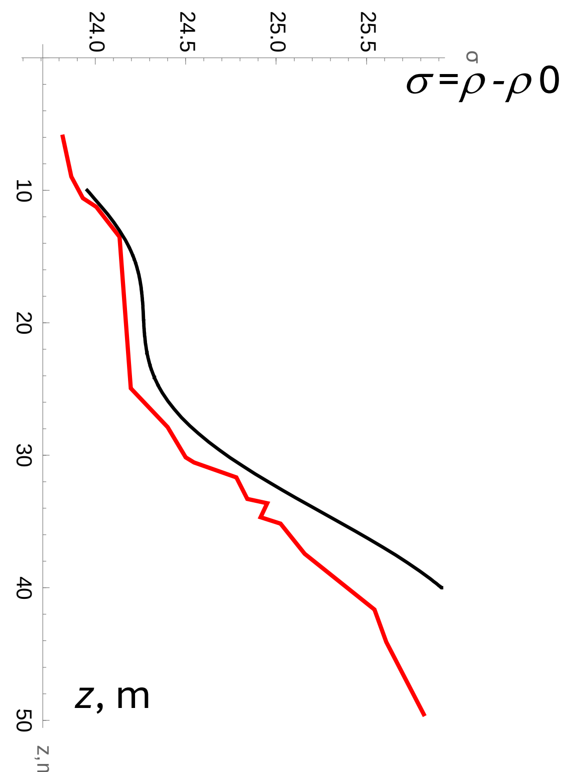
Profiles of velocity and density near soliton maximum



Vertical profiles of current velocity (left) and density (right) in red and polynomial adjustments in the cross-section near maximum (marked by the circle above)



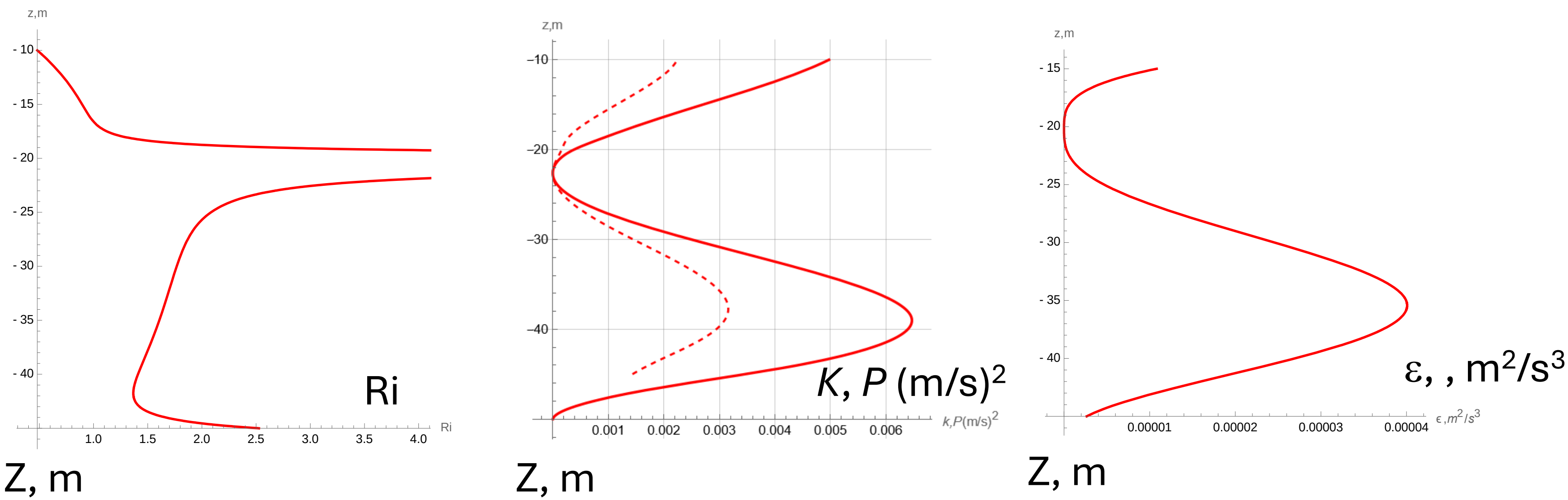
Horiz. velocity profile



Density profile

Quasi-static turbulence distribution

Left to right: Profiles of Richardson number, Kinetic and potential energies, and Dissipation rate



Richardson no.

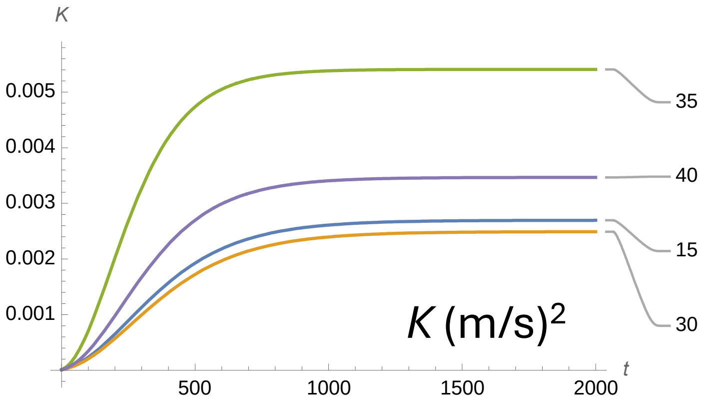
Kinetic (solid) and potential (dashed) energies

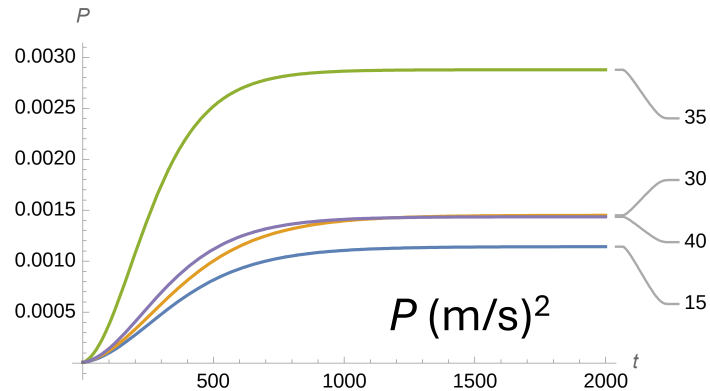
TKE dissipation rate

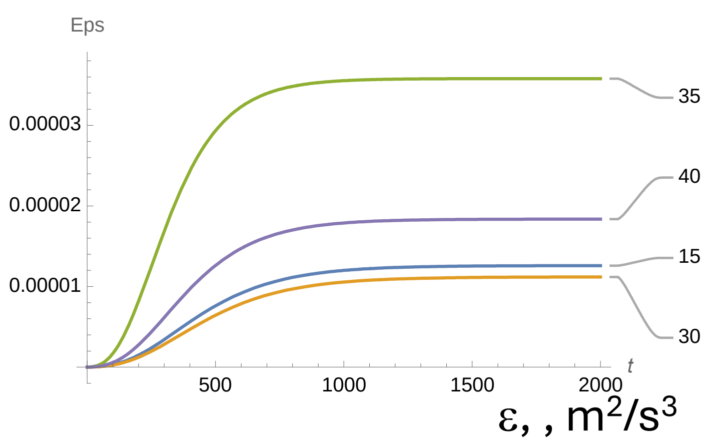
Maximum of dissipation rate is at same depth (about 35 m) as in Moum et al. (2003), but ε here is by an order higher (10^{-5} vs 10^{-6}). This is because the process is non-stationary.

Dynamics of turbulent energy

Evolution of kinetic and potential energies and TKE dissipation rate at different depths

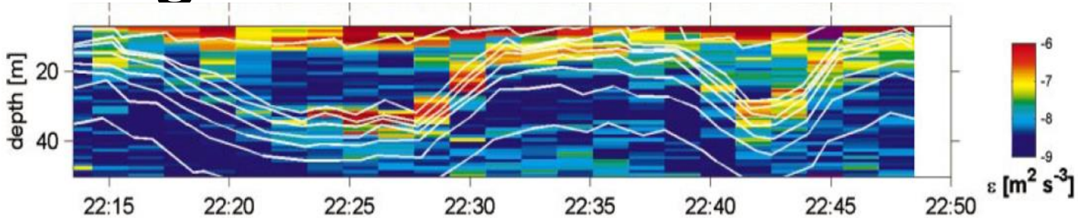


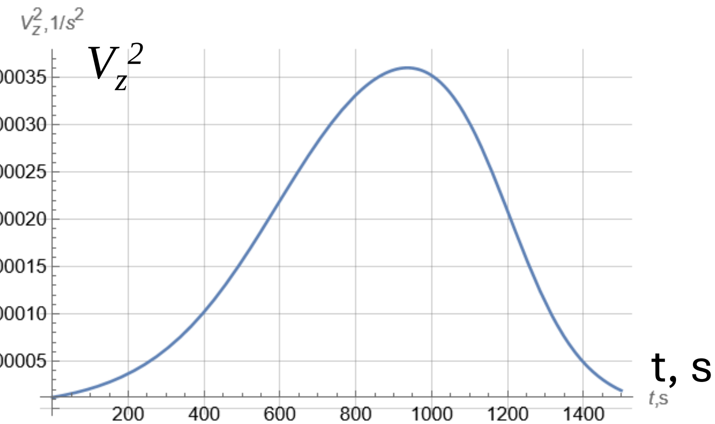




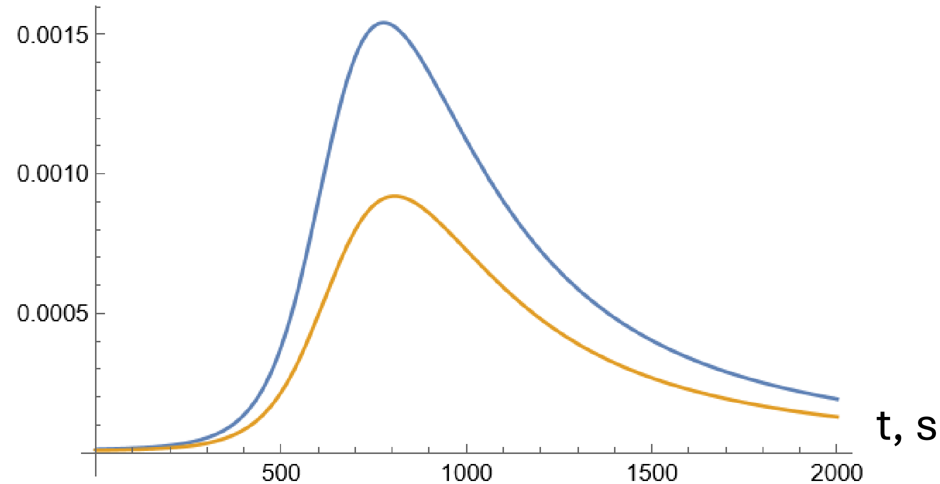
Transient time (10-15 minutes) is comparable with the half-duration of the soliton so that the process is not stationary.

Variation along the soliton maximum

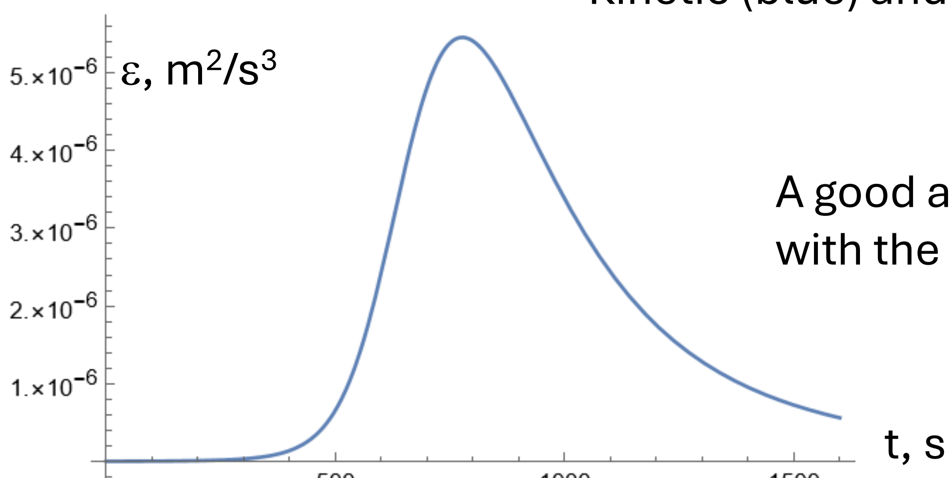




Horiz. velocity shear



Kinetic (blue) and potential (yellow) energies



Turb. dissipation rate

A good agreement
with the experiment

Concluding remarks

- Возмущения флуктуаций плотности в стратифицированной среде, связанные с потенциальной энергией турбулентности, могут радикально менять условия усиления и поддержания турбулентности за счет энергии сдвиговых течений и внутренних волн и в результате баланс энергии в верхнем слое океана.

- К настоящему времени нам известно очень небольшое число публикаций, содержащих достаточно натурных данных для сравнения с теорией. Мы надеемся найти больше.

- Ряд результатов в этой области получен для пограничного слоя атмосферы С.С. Зилитинкевичем (отчасти совместно с автором).

- Предполагается развить теорию для ионосферной (плазменной) турбулентности, в частности, используя работы С.С.Моисеева.