About magnetic filaments in the solar convective zone: new results

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OUTLINE

- Motivation: coherent structures for frozen-in-fluid fields and their amplification
- Analog of the VLR and its compressibility
- Parameters of the solar convective zone
- Formation of magnetic filaments in convective zone of the Sun
- Reductions to the free surface

Papers

- E.A. Kuznetsov, E.A. Mikhailov, *Notes about collapse in magnetohydrodynamics*, ZhETF, **158**, 1 (2020); arXiv:3276237 [physics.geo-ph].
- E.A. Kuznetsov, E.A. Mikhailov, *Magnetic filaments: formation, stability, and feedback*, Special Issue "Numerical and Analytical Study of Fluid Dynamics", Mathematics, **12**, 677 (1-14) (2024); arXiv:2402.16989 [astro-ph.SR].

Motivation

Collapse as a process of the singularity formation for smooth initial conditions represents one of the key issues for understanding nature for both hydrodynamic turbulence and MHD turbulence. The Kolmogorov-Obukhov theory of developed hydrodynamic turbulence at large Reynolds numbers, $Re \gg 1,$ in the inertial interval predicts the divergence of vorticity fluctuations $\langle \delta\omega \rangle$ at small ℓ as $\ell^{-2/3}$, which indicates the connection of Kolmogorov turbulence with collapse.

Motivation

In 3D Euler hydrodynamics we numerically and analytically (see, our review paper: D.S. Agafontsev, E.A. Kuznetsov, A.A. Mailybaev, E.V. Sereshchenko, *Compressible vortical structures and their role in the hydrodynamical turbulence onset*, UFN **192**, 205-225 (2022) [Physics Uspekhi, **65** 189 - 208 (2022)]) showed an exponential increase in time for the vorticity ω inside the pancake-type vortex structures. The formation of such structures is possible because of frozenness of 3D vorticity. Due to this property the continuously distributed vortex lines occur compressible. In this case maximal values of vorticity are evaluated proportionally to their widths as $\ell^{-2/3}$. This process can be considered as *folding* like breaking for gases.

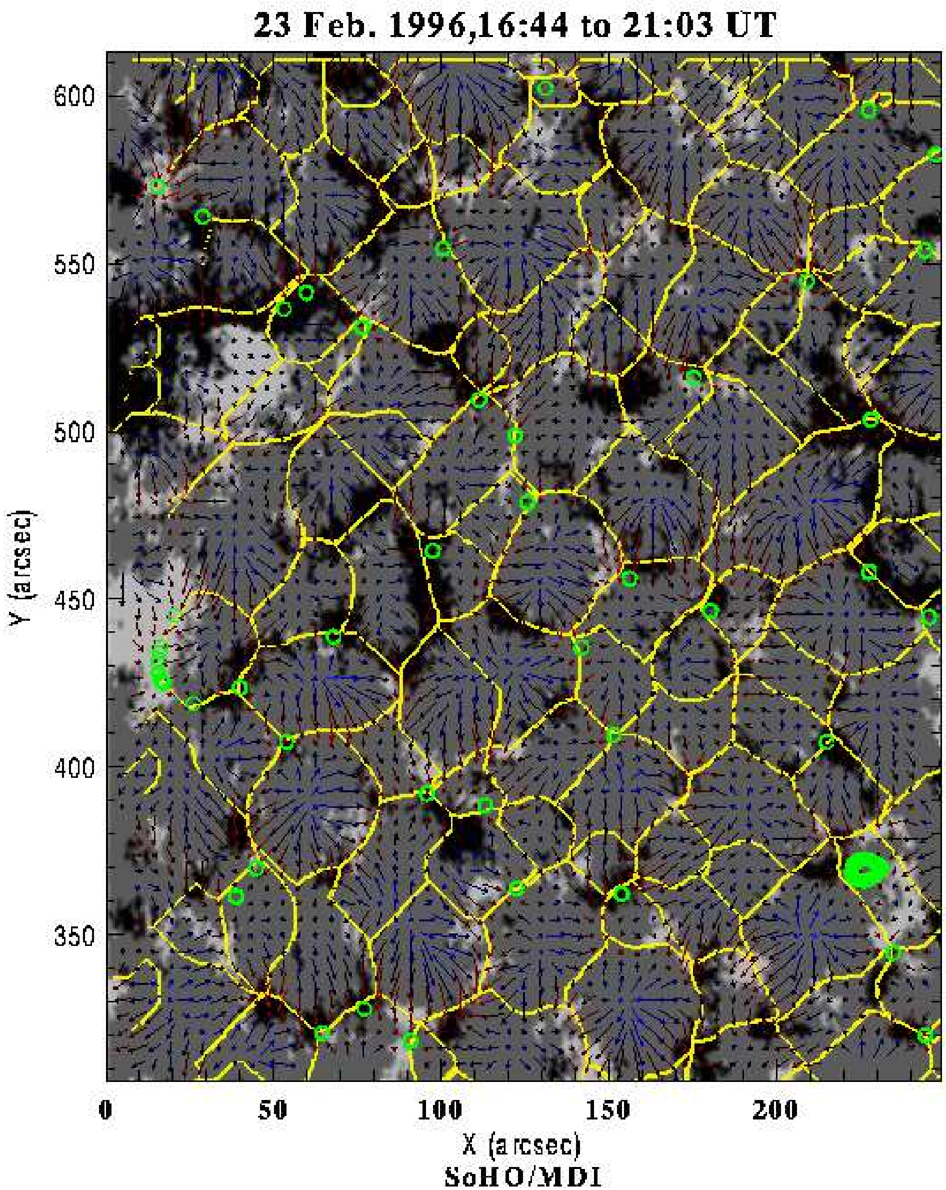
Motivation

In MHD at high values of magnetic Reynolds numbers, $Re_m \gg 1$, the magnetic field is also a frozen-in field. Therefore one can expect, that the exponential in time growth also should be observed due to compressibility of magnetic field lines. Such situation is realized in the convective zone of the Sun that leads to the magnetic field filamentation. This was first addressed attention in the Parker's pioneer work in 1963.

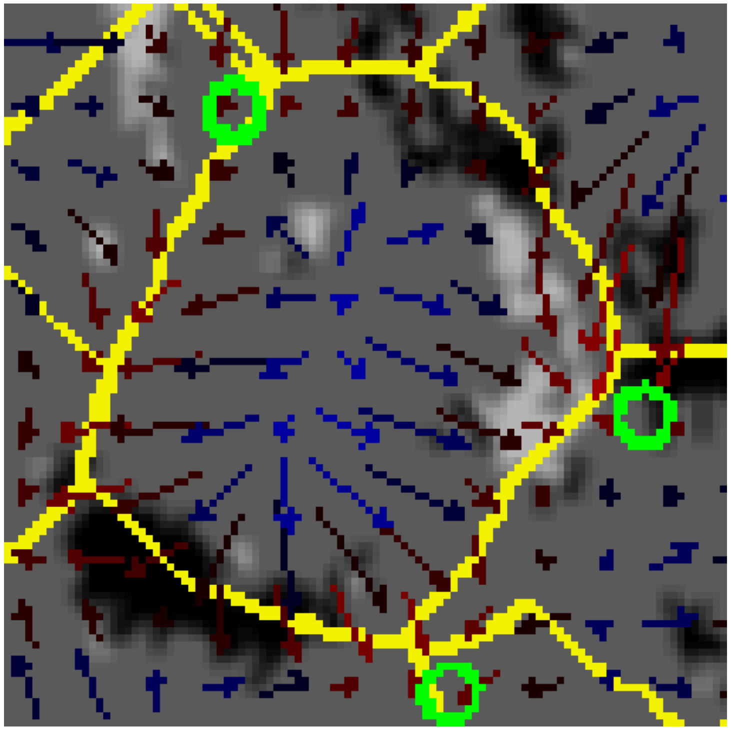
Formation of magnetic filaments in convective zone of the Sun

Now we consider the problem of generation of strong magnetic fields in MHD due to the folding mechanism predicted in K., T. Passot and P.L. Sulem (Physics of Plasmas, **11**, 1410-1415 (2004)). On our opinion, the formation of magnetic filaments in the convective zone of the Sun can be explained by this mechanism.

Formation of magnetic filaments in convective zone of the Sun

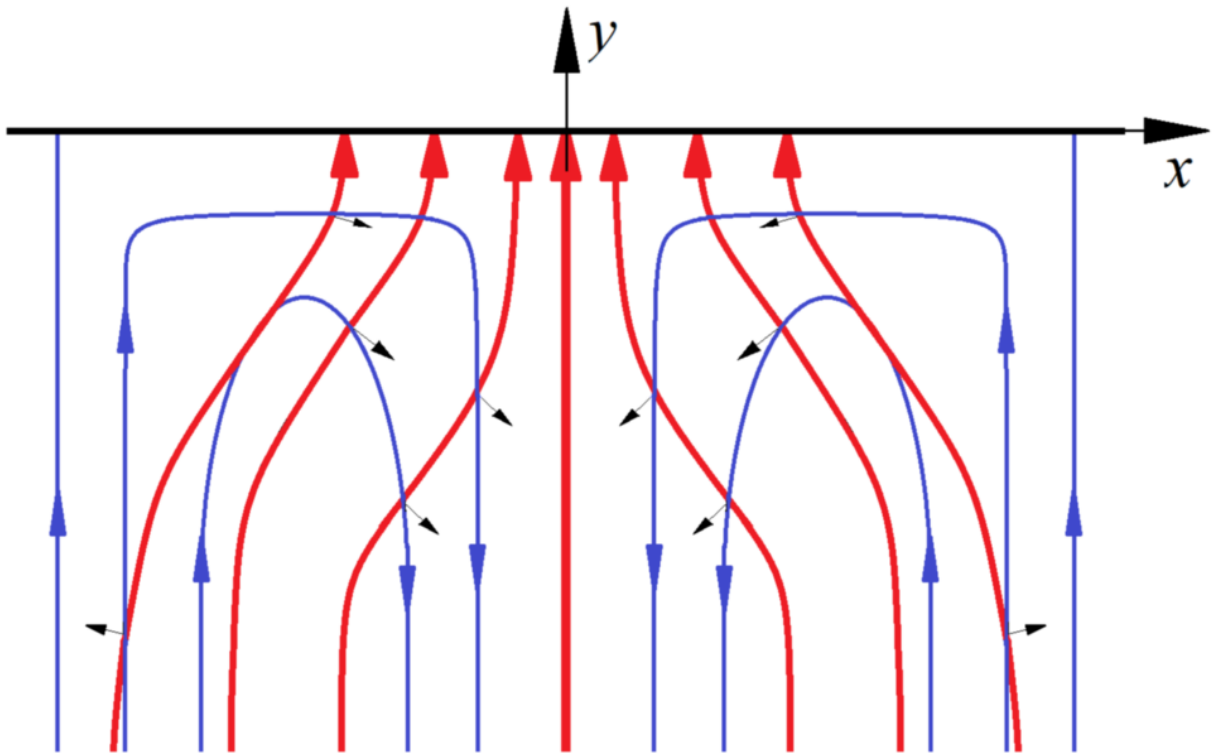


Formation of magnetic filaments in convective zone of the Sun



SOHO magnetogram overlaid with lines of convergence of the horizontal flow and with green dots showing the convergence points. The measured flow is shown as colored arrows, red for inferred downflow and blue for inferred upflow. The field is shown light grey for positive fields and dark for negative fields. Only field above the background noise is shown.

Formation of magnetic filaments in convective zone of the Sun



Streamlines (blue), magnetic lines (red), normal velocity (black)

Parameters of the Sun convective zone

- For the convection zone $\rho\mathbf{v}^2/2 \gg \mathbf{B}^2/(8\pi)$ (their ratio is about 10^5) where density $\rho =$ const and div $\mathbf{v} = 0$. Therefore the fluid velocity can be considered as a given vector field. Near the boundary with photosphere (the beginning of the Sun atmosphere) $\rho \sim 10^{-5}\, g/sm^3$ and $v \sim 500\, m/s$. The mean solar magnetic field is about a few gauss (for estimate we take $B_0 = 10\, G$).
- The magnetic Reynolds number $Re_m \sim 10^6 - 10^8$ that allows one to neglect magnetic viscosity. This means that the magnetic field in this approximation can be considered as frozen-in one:

$$\frac{\partial \mathbf{B}}{\partial t} = \text{rot}(\mathbf{v} \times \mathbf{B}), \ \operatorname{div} \mathbf{v} = 0.$$

Formation of magnetic filaments in convective zone of the Sun

- The induction equation for $\mathbf{B}$ in terms of magnetic potential A in the 2D case is written as
  $\frac{\partial}{\partial t}A + (\mathbf{v} \cdot \nabla)A = 0$.
  The isolines of A coincide with magnetic field lines.
  Further we will assume that at $t = 0$ the magnetic field is const: $\mathbf{B} = (0, B_0)$.
- Consider simplest 2D stationary convective cell given by stream function $\psi = \sin x \cdot \sin y$ (of the roll type), which gives velocity components
  $v_x = -\sin x \cdot \cos y$, $v_y = \cos x \cdot \sin y$.
  For such flows the equation for A admits application of the method of characteristics.

Formation of magnetic filaments in convective zone of the Sun

The equations for characteristics are the Hamiltonian ones with ψ as the Hamiltonian:

$$\frac{dx}{dt} = -\frac{\partial \psi}{\partial y}, \quad \frac{dy}{dt} = \frac{\partial \psi}{\partial x}$$

with the initial conditions $\mathbf{r}|_{t=0} = \mathbf{a}$.

If one considers a small vicinity near the point $x = y = 0$ where we should expect the magnetic field amplification and the formation of the magnetic filament then the stream function should be written as $\psi = xy$ with the initial conditions $\psi = a_x a_y$. Then the obtained equations are turned out linear that give:

$$x = a_x e^{-t} \text{ and } y = a_y e^t.$$

For the initial condition for $A = -B_0 a_x$ it is easily to get that

$$B_x = 0, \; B_y = B_0 e^t.$$

These analytical results are confirmed by simulations.

Formation of magnetic filaments in convective zone of the Sun

In the general situation for 2D flows filamentation takes place in the hyperbolic points where

$$\psi_{xx}\psi_{yy} - \psi_{xy}^2 < 0,$$

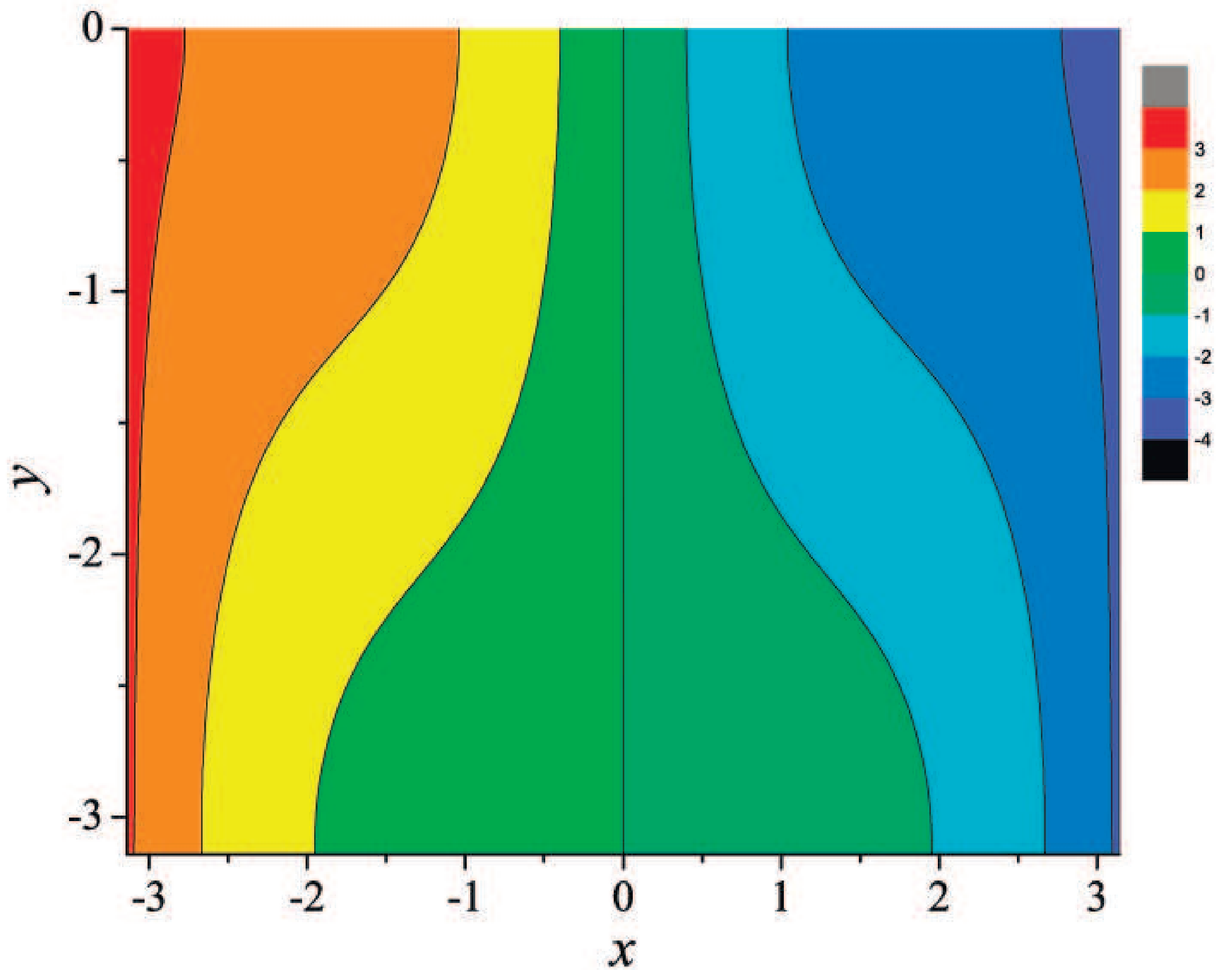
respectively, a region where this criterion holds is called hyperbolic. In the opposite case,

$$\psi_{xx}\psi_{yy} - \psi_{xy}^2 > 0,$$

such points and regions are called elliptic. In the latter case the magnetic field rotates around such points.

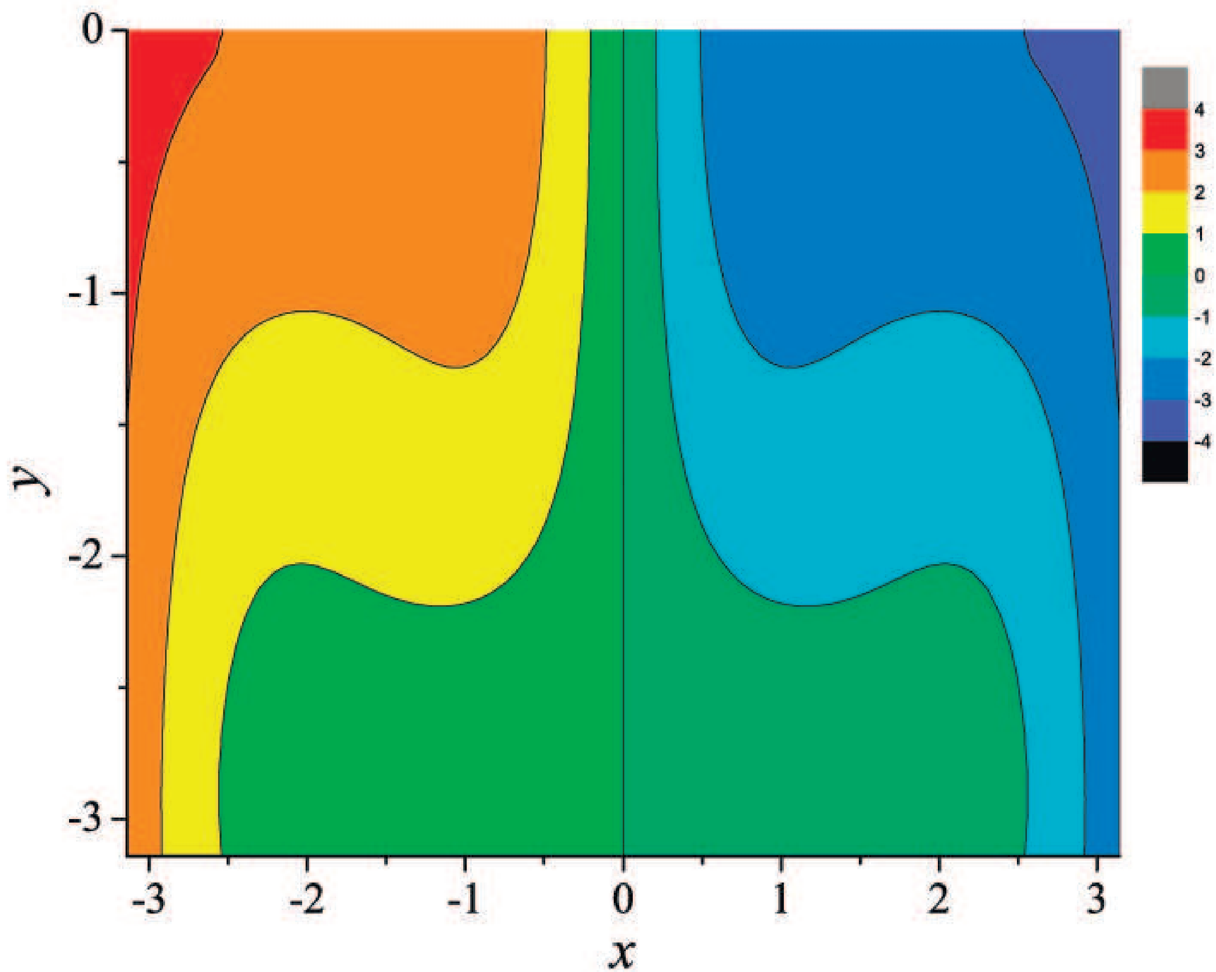
Formation of magnetic filaments in convective zone of the Sun

The profile of magnetic potential A in the region $-\pi \leq x \leq \pi$, $-\pi \leq y \leq 0$ at $t = 1$ for magnetic viscosity $\eta = 0$.



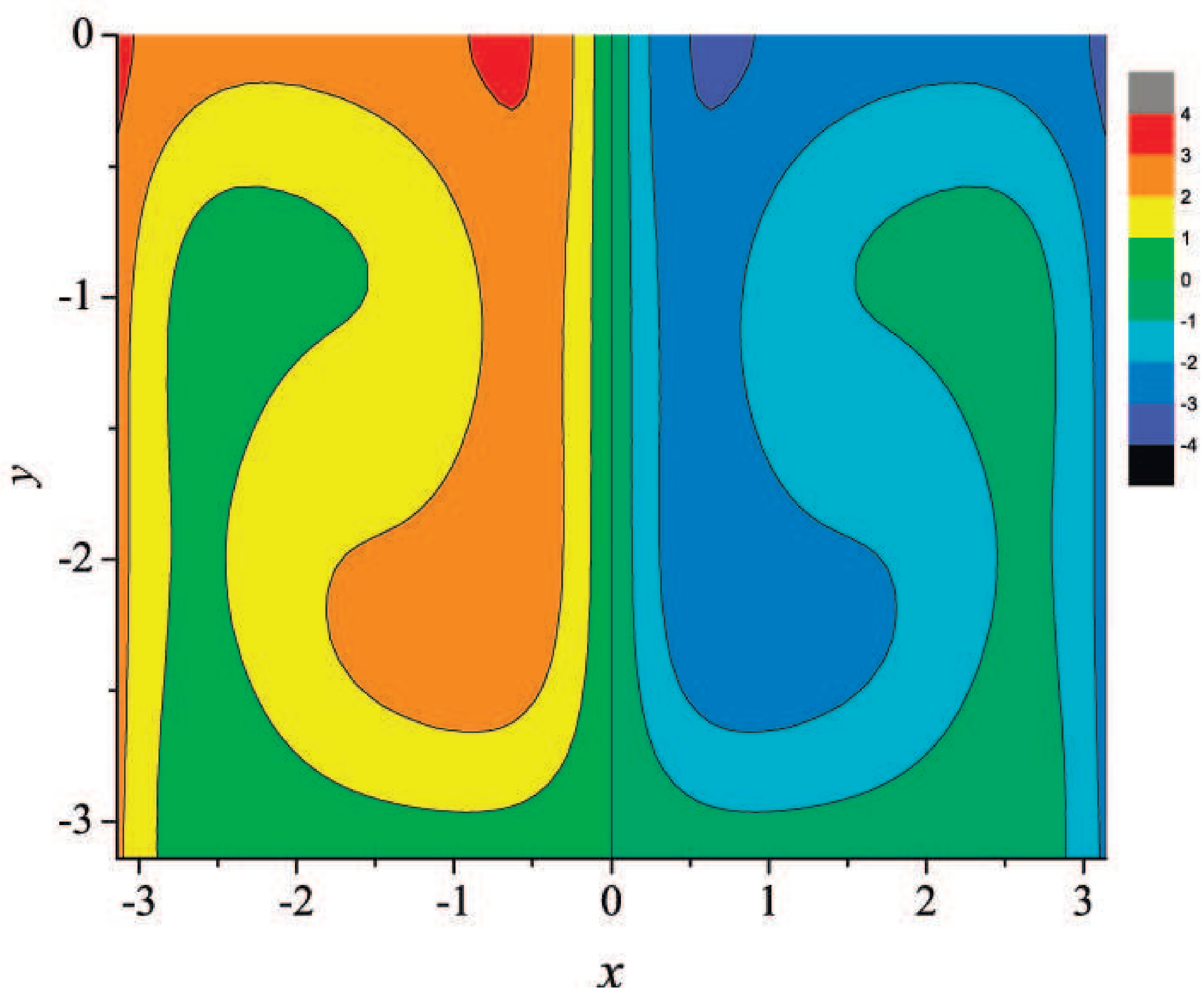
Formation of magnetic filaments in convective zone of the Sun

The profile of magnetic potential A at $t = 2$ for $\eta = 0$.



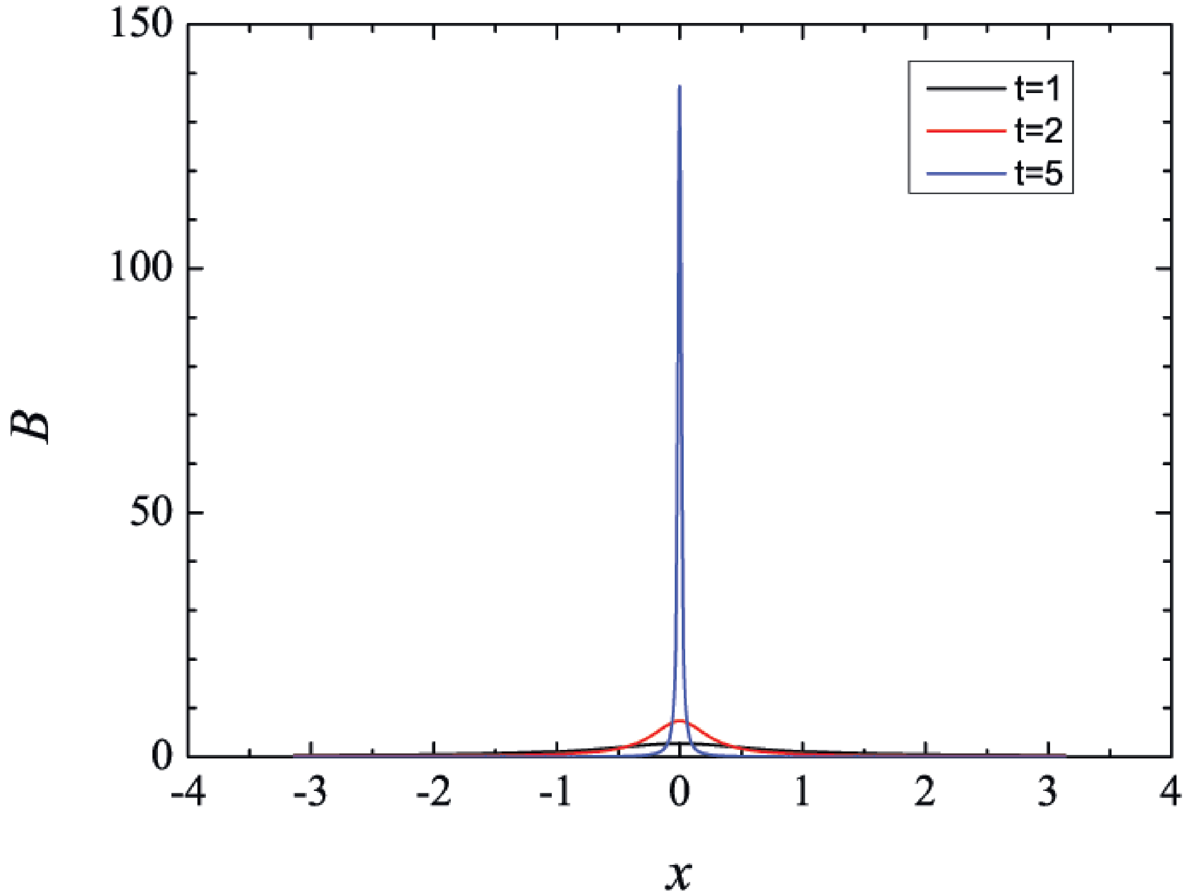
Formation of magnetic filaments in convective zone of the Sun

The profile of magnetic potential A at $t = 5$ for $\eta = 0$.



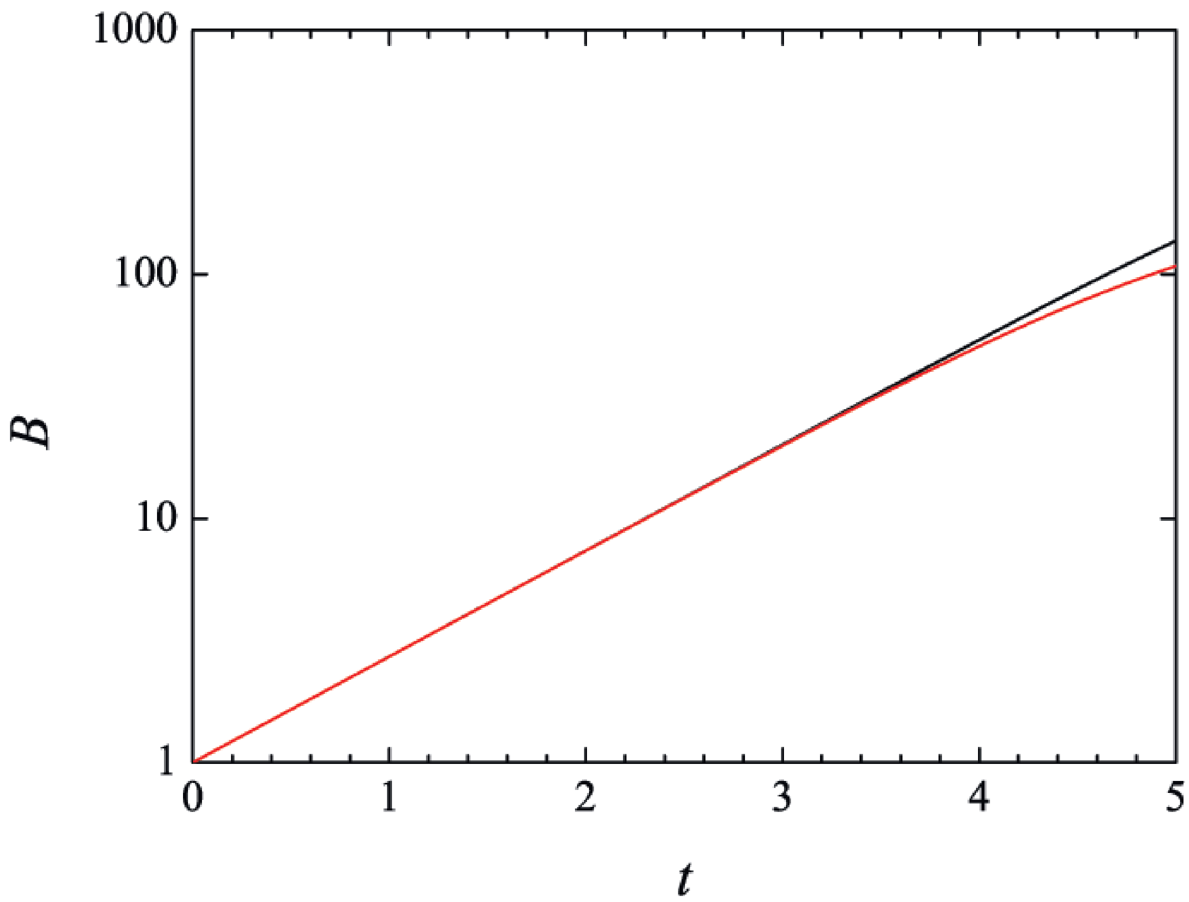
Formation of magnetic filaments in convective zone of the Sun

The behavior of B_y along free surface $y = 0$ for magnetic viscosity $\eta = 0$.



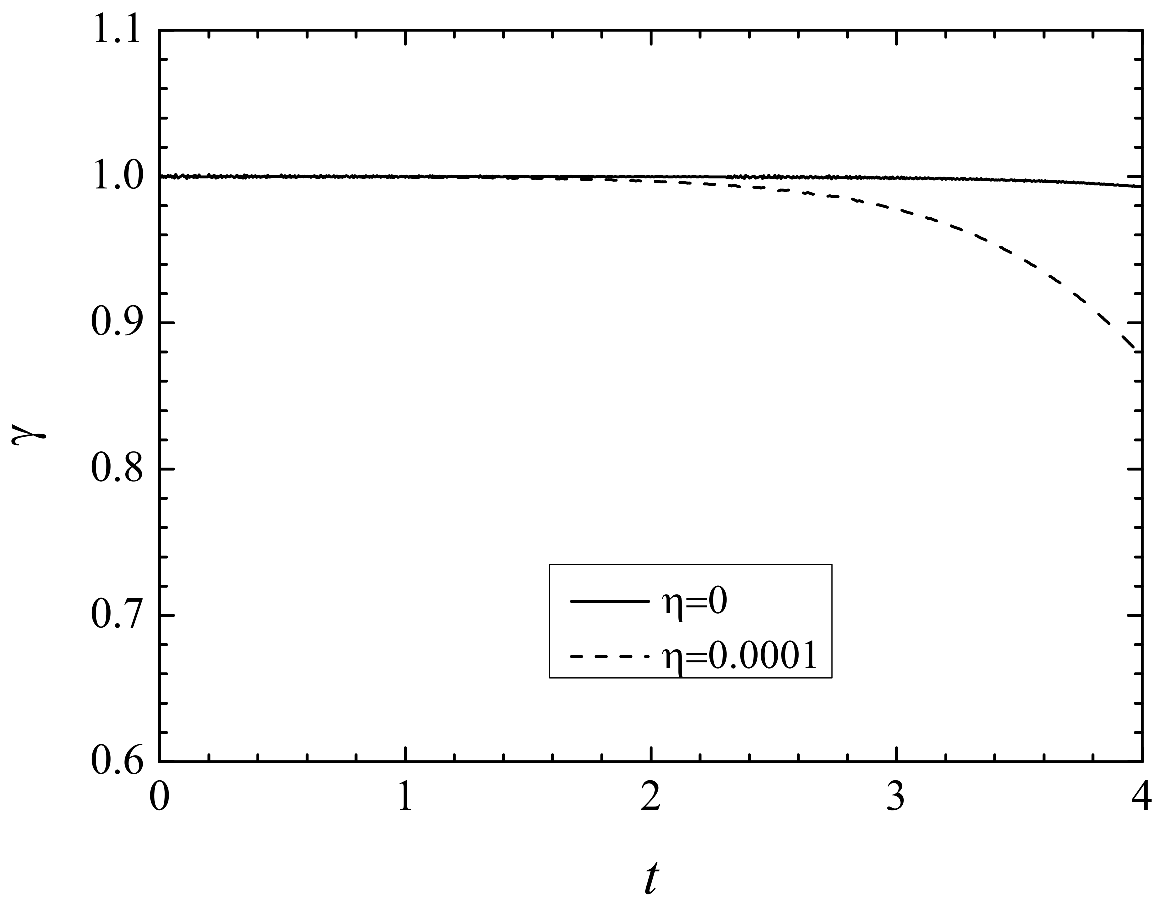
Formation of magnetic filaments in convective zone of the Sun

The behavior of maximal B at $x = y = 0$: black line for zero magnetic viscosity $\eta = 0$, red line for $\eta = 0.0001$.



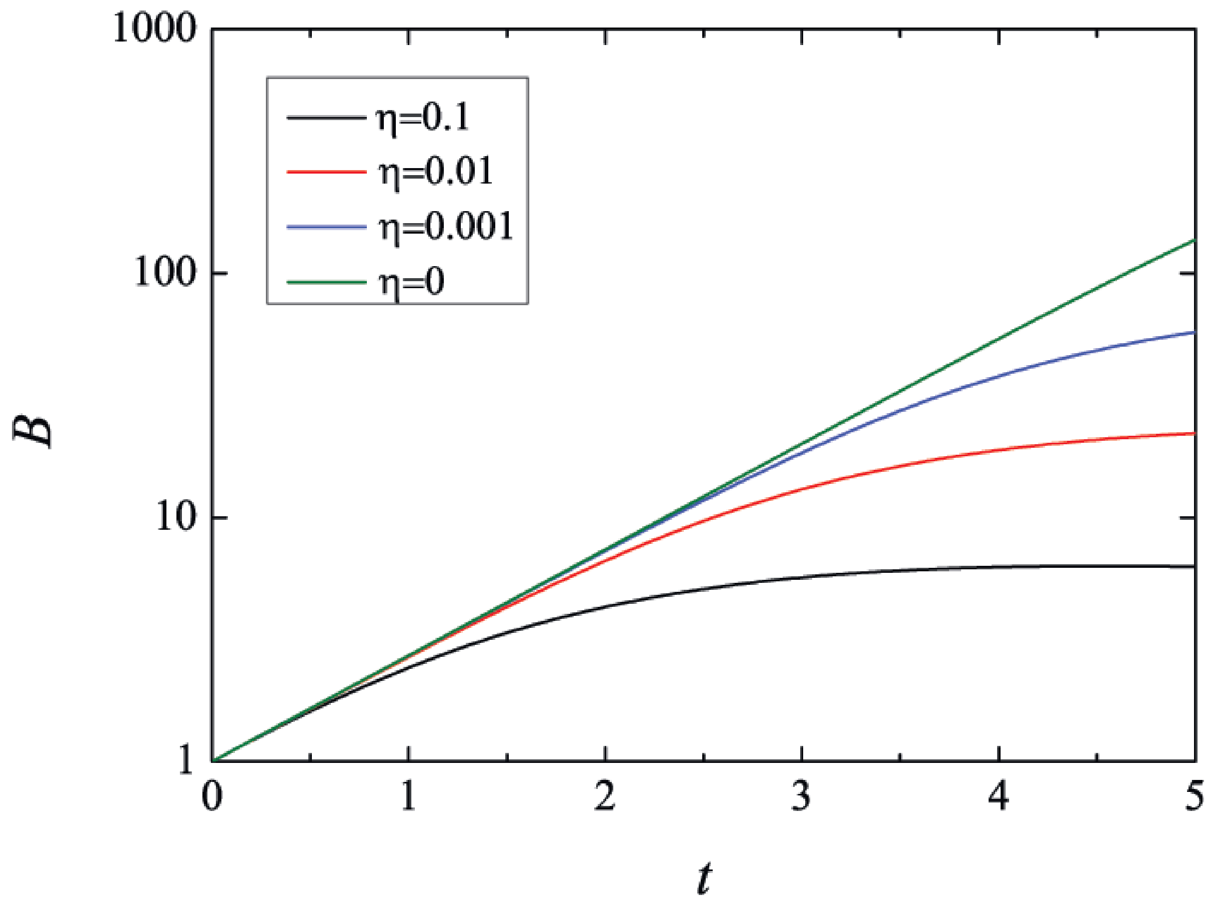
Formation of magnetic filaments in convective zone of the Sun

Growth rate $\gamma = d\ln B_{max}/dt$ at $x = y = 0$.



Formation of magnetic filaments in convective zone of the Sun

The temporal behavior of maximal B for different η . Viscosity destroys the frozenness of magnetic field that results in saturation.



Formation of magnetic filaments in convective zone of the Sun

Based on the magnetic field flux conservation it is possible to get the saturated field estimate:

$$B_{sat} \sim Re_m^{1/2} B_0$$

For $B_0 = 10\,G$ and $Re_m = 10^6$ we have $B_{sat} \sim 10^4\,G$. For such large values the feedback influence of the growing magnetic field on the convection velocity is seemed large, but however because the magnetic field in filament is perpendicular to its gradient this influence occurs not too essential.

In the 3D case, the mechanism of filamentation remains the same like in 2D. The difference with 2D is connected with magnetic filaments become flatten relative to the interface between cells.

Reductions

In the second part of this talk, we will show how the problem of the formation of magnetic filaments in the convective zone of the Sun can be qualitatively studied based only on an analysis of the dynamics of the free surface of convective cells, i.e., reducing the dimension of the problem. This makes it possible to explain the formation of magnetic filaments in the vicinity of the interfaces between convective cells, i.e., in areas of downward flows, which act as specific attractors of the magnetic field. The growth of the magnetic field in the filaments occurs due to the frozenness of the field. It is important that this result does not depend on the specific structure of the convective cell. A numerical experiment confirms this analytical concept.

Reductions

The SOHO experimental observations show that the top convective surface is almost flat. We can ignore the surface deviations and can put the normal velocity component $v_n = 0$ on the free surface Γ. Evidently that in this case on Γ $\nabla_\perp v_n = 0$ also. With account of incompressibility condition $\mathrm{div}\,\mathbf{v} = 0$ equation for the normal component of $\mathbf{B}$ on Γ at $Re_m >> 1$ can be written as follows

$$\frac{\partial B_n}{\partial t} + (\mathbf{v}_\perp \cdot \nabla_\perp) B_n = -B_n(\nabla_\perp \cdot \mathbf{v}_\perp).$$

For 2D flows this equation is rewritten as

$$\frac{\partial B_y}{\partial t} + v_x \frac{\partial B_y}{\partial x} = -B_y \frac{\partial v_x}{\partial x}.$$

Reduction from 2D flow to 1D flow

Thus, dynamics of the normal magnetic field on the free surface is defined only by the tangent velocity on Γ and does not depend on the velocity structure inside the convective volume.

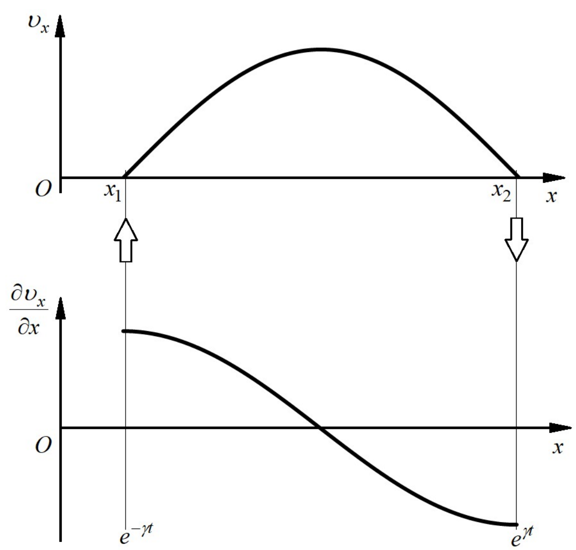
This 1D equation (as well as 2D equation) is solved by the method of characteristics. Along the characteristics $dx/dt = v_x$ the magnetic component B_y obeys the equation

$$\frac{dB_y}{dt} = -B_y \frac{\partial v_x}{\partial x}.$$

Thus, B_y is transferred by the advection flow v_x with simultaneous changing value of B_y. For the stationary flow at the center of the convective cell $-\partial v_x/\partial x$ will be negative (note that in this point $v_x = 0$) and B_y will decrease exponentially with time.

Reduction from 2D flow to 1D flow

Scheme of the generation of the magnetic field. Upper plot shows the profile of the velocity component parallel to the surface $y = 0$, lower one is its spatial derivative. Arrows indicate centers of upward (source) and downward (sink) flows, respectively.



Reduction from 2D flow to 1D surface flow

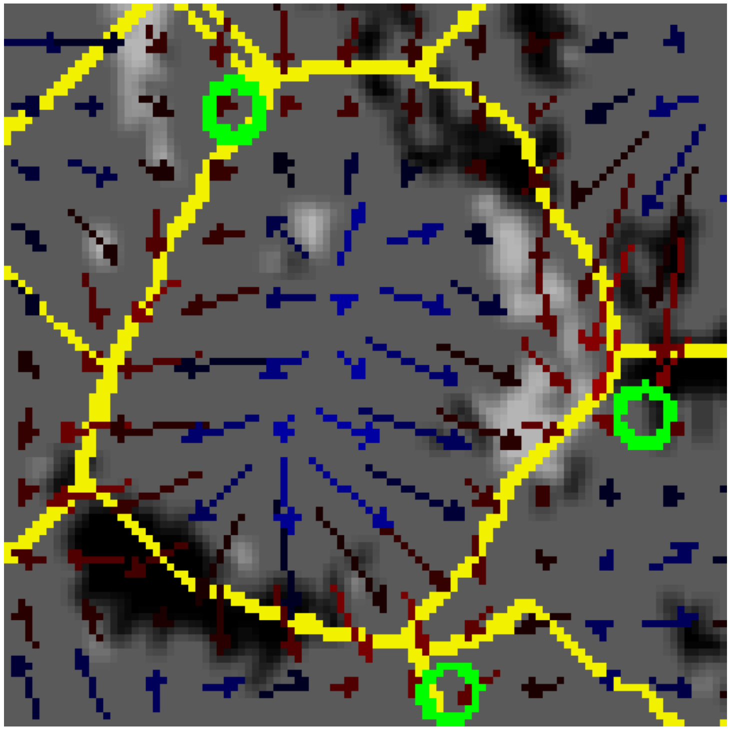
Consider now how the transverse magnetic field component B_x behaves in time. In this case it is enough to analyze its behavior at the interface between convective cells. There at $y = 0$ the velocity component $v_y = 0$ and it will increase with growth of $-y$. Then the derivative $\partial v_y/\partial y > 0$ will positive that in accordance with previous consideration provides exponential decrease of the B_x value. Simultaneously the localization region will grow exponentially. Thus, this analysis leads us to the following conclusion. Formation of magnetic filaments represents very robust process, it is independent on the structure of the velocity field in the convection bulk.

Reduction from 2D flow to 1D surface flow

We have checked numerically for two-dimensional computation of the convective cells with different aspect ratio between horizontal and vertical scales of the that the maximum of growth rate γ always takes place at the hyperbolic point and depends on the behavior of the v_x there.

Reduction from 3D flow to 2D surface flow

Consider the SOHO data.



Reduction from 3D flow to 2D surface flow

Hence one can see convergence of the horizontal flow. In its center the velocity $\mathbf{v}_\perp = 0$ and respectively near the center $\mathrm{div}\,\mathbf{v}_\perp$ is positive (source). When we arrive at the boundary the velocity $\mathbf{v}_\perp$ vanishes also but $\mathrm{div}\,\mathbf{v}_\perp$ becomes negative (sink) that leads to the magnetic filaments formation there. It is worth noticed that analysis of many other cells showed that the magnetic filaments are concentrated near interfaces between convective cells and filaments are very rare at the cell centers.

Conclusion

- Convective cells are responsible for the formation of magnetic filaments in convective zone of the Sun.
- The formation of magnetic filaments is connected with compressibility of the magnetic field. This process in time has exponential character.
- Directions of magnetic filaments are correlated with convective down flow. This flow plays a role of an attractor for the magnetic field. Filaments are concentrated around interfaces of convective cells.

THANKS FOR YOUR ATTENTION