# STUDIES OF THE MARTIAN BOW SHOCK RESPONSE TO THE VARIATION OF THE MAGNETOSPHERE DIMENSIONS ACCORDING TO TAUS AND MAGMA MEASUREMENTS ABOARD THE PHOBOS 2 ORBITER 

M. I. Verigin,* G. A. Kotova,* A. P. Remizov,* N. M. Shutte,*<br>K. Schwingenschuh,** W. Riedler,** T.-L. Zhang,** H. Rosenbauer,*** K. Szegö, ${ }^{\dagger}$ M. Tatrallyay ${ }^{\dagger}$ and V. Styazhkin ${ }^{\ddagger}$<br>* Space Research Institute of Russian Academy of Sciences, Profsoyuznaya 84/32, Moscow 117810, Russia<br>** Space Reserach Institute, Inffeldgasse $12,8010 \mathrm{Graz}$, Austria<br>*** Max-Planck-Institut für Aeronomie, D-37191 Katlenburg-Lindau, Germany<br>$\dagger$ KFKI Research Institute for Particle and Nuclear Physics, 1525 Budapest, P.O. Box 49, Hungary<br>$\ddagger$ Institute for Terrestrial Magnetism and Radio Waves Propagation, Troitzk, Moscow Region 142092. Russia


#### Abstract

A self-consistent quantitative model is presented which describes the planetary bow shock motion and its shape variation due to variations of the external plasma flow parameters and the magnetopause shape. This model is applied to the analysis of the Martian bow shock motions and for the explanation of its unusual properties.


(c) 1997 COSPAR. Published by Elsevier Science Ldd.

## INTRODUCTION

Multiple crossings of the Martian bow shock by the Phobos 2 orbiter disclosed unusual properties of this boundary as compared to other planetary shocks. The terminator position of the bow shock turned to be very weakly dependent on the solar wind ram pressure $\rho \mathrm{V}^{2}$ (Schwingenschuh et al, 1992; Verigin et al., 1993) resembling similar dependence at nonmagnetic Venus (Tatrallyay et al., 1983). Meanwhile the location of the Martian shock at terminator was subjected to very large temporal variations, specific for the planets where bow shocks are formed by a 'soft' obstacle produced by intrinsic dipole planetary field. These variations, however, did not obscure the dependence of the Martian bow shock terminator position on the angle between the shock normal and interplanetary magnetic field $\vartheta_{b n}$ similar to the dependence revealed at Venus (Zhang et al., 1991).

In order to explain unusual properties of the Martian bow shock Verigin et al. (1993) assumed the stable position of the magnetopause subsolar region, and later they developed a quantitative model of the Martian magnetopause for different $\rho V^{2}$ (Verigin et al., 1996). Specific features of this model are the 'stagnation' of the subsolar magnetopause for $\rho^{2} \geq 6 \times 10^{-9} \mathrm{dyn} \mathrm{cm}^{-2}$ and the variation of the magnetopause shape as a function of the solar wind ram pressure. In the present paper we will try to compare the observed positions of the Martian bow shock with positions calculated using the model magnetopause shape and solar wind parameters observed in individual Phobos 2 orbits. An empirical bow shock model will be presented which provides possibility of fast and reasonably accurate calculation of the bow shock position at any zenith angle for different magnetopause shapes, $\vartheta_{\mathrm{bn}}$, and a wide range of upstream sonic $\mathrm{M}_{\mathrm{s}}$ and Alfvenic $\mathrm{M}_{\mathrm{a}}$ Mach numbers.

## APPROACH TO THE BOW SHOCK MODELING AND COMPARISON WITH PREVIOUS MODELS

Modeling of the planetary shocks began from the calculations of the HD flow around the obstacle with a shape resembling the geomagnetosphere $\left(\mathrm{R}_{0} / \mathrm{r}_{0} \approx 1.26\right.$, see definition of space variables in Figure 1 ) for several values of $\mathrm{M}_{\mathbf{s}}$ and specific heats ratio $\gamma$ (Spreiter et al., 1966). These bulky calculations combined with earlier aerodynamic empirical idea that $\Delta$ is a function of the fluid density compression across the bow shock


Fig. 1. Definition of space variables: $r_{0}, r_{s}$ are distances to the obstacle (magnetopause) and shock, respectively, $\mathrm{R}_{\mathbf{0}}, \mathrm{R}_{\mathrm{s}}$ are proper curvature radii, $\Delta$ is the bow shock stand off distance.

$$
\begin{equation*}
\varepsilon=\rho_{1} / \rho_{2}=\left[(\gamma-1) \mathrm{M}_{\mathrm{s}}^{2}+2\right] /\left[(\gamma+1) \mathrm{M}_{\mathrm{s}}^{2}\right] \tag{1}
\end{equation*}
$$

(subscripts 1, 2 correspond to flow regimes upstream and downstream the shock, respectively) lead Spreiter et al. (1966) to the following linear relation:

$$
\begin{equation*}
\Delta / r_{0}=1.1 \cdot \varepsilon, \quad 5<M_{s}<\infty . \tag{2}
\end{equation*}
$$

Taking into account that $\Delta \rightarrow \infty$ when $\mathrm{M}_{\mathrm{s}} \rightarrow 1 \mathrm{Eq} .2$ was intuitively modified by Farris \& Russell (1994) to the following one:

$$
\begin{equation*}
\Delta / \mathrm{r}_{0}=1.1 \cdot \varepsilon \cdot \mathrm{M}_{\mathrm{s}}^{2} /\left(\mathrm{M}_{\mathrm{s}}^{2}-1\right) \tag{3}
\end{equation*}
$$

though with uncertain limits of applicability. Improvement of computers permitted to realize MHD calculations of the flow around magnetosphere ( $\mathrm{R}_{0} / \mathrm{r}_{0} \approx 1.47$, Cairns \& Lion, 1995) but again for a limited number of $\mathrm{M}_{\mathrm{s}}, \mathrm{M}_{\mathrm{a}}$ and $\vartheta_{\mathrm{bn}}$ combinations. The following linear relation was introduced for the approximation of these results:

$$
\begin{equation*}
\Delta / r_{0}=3.4 \cdot \varepsilon-0.6, M_{s}=7.6,1.4<M_{a}<\infty \tag{4}
\end{equation*}
$$

where $\varepsilon$ now is the real root of the following cubic equation (e.g., Zhuang \&Russell, 1981):

$$
\begin{align*}
& \varepsilon^{3}-\left(\frac{\gamma-1}{\gamma+1}+\frac{\gamma+(\gamma+2) \cos ^{2} \vartheta_{b n}}{(\gamma+1) M_{a}^{2}}+\frac{2}{(\gamma+1) M_{s}^{2}}\right) \varepsilon^{2}+  \tag{5}\\
& +\frac{1}{(\gamma+1) M_{a}^{2}}\left(\gamma\left(1+\cos ^{2} \vartheta_{b n}\right)-2+\cos ^{2} g_{b n}\left(\frac{\gamma+1}{M_{a}^{2}}+\frac{4}{M_{s}^{2}}\right)\right) \varepsilon-\frac{\cos ^{2} g_{b n}}{(\gamma+1) M_{a}^{4}}\left(\gamma-1+\frac{2 \cos ^{2} g_{b n}}{M_{s}^{2}}\right)=0 .
\end{align*}
$$

Calculations of the gasdynamic flow around different bodies had a long history prior to they were applied to the flow around magnetosphere. Among multiple approximations available it is worth to present expressions of Minailos (1973) and Stulov (1969) for $\Delta$ and $R_{s}$, respectively:

$$
\begin{equation*}
\Delta / R_{0}=\varepsilon\left(0.76+1.05 \varepsilon^{2}\right), \quad 1.5<M_{s}<\infty ; \quad R_{s}=\Delta \cdot(1+\sqrt{8 \varepsilon / 3}) / \varepsilon, \quad M_{s} \geq 3 \tag{6}
\end{equation*}
$$

and results of analytical studies by Shugaev (1964) of asymptotic behavior of $R_{s}$ and $\Delta$ when $M_{s} \rightarrow 1$ :

$$
\begin{equation*}
\mathrm{R}_{\mathrm{s}} \sim\left(\mathrm{M}_{\mathrm{s}}-1\right)^{-5 / 3}, \quad \Delta \sim\left(\mathrm{M}_{\mathrm{s}}-1\right)^{-2 / 3} \tag{7}
\end{equation*}
$$

From the boundary condition specific to the symmetry axis of the curved shock (e.g., Biermann et al., 1967) one can deduce that both $R_{s}$ and $\Delta$ can naturally be approximated as a function of $\varepsilon^{\prime} \equiv \varepsilon /(1-\varepsilon)$. Omitting details we will present expressions for $\Delta$ and $\mathrm{K}_{\mathrm{s}}$ below, which approximately coincide with Eqs. 6 for sufficiently large $\mathrm{M}_{\mathrm{s}}$, have proper asymptotic behavior (Eq. 7) when $\mathrm{M}_{\mathrm{s}} \rightarrow 1$, and reasonably describe results of gasdynamic experiments (see, e.g., data summarized by Belotserkovsky et al., 1967) for intermediate values of $\mathrm{M}_{3}$ :

$$
\begin{equation*}
\Delta / \mathrm{r}_{0}=\left(\mathrm{R}_{0} / \mathrm{r}_{0}\right) \cdot\left(\varepsilon^{\prime} /\left(1.87+0.86 / \varepsilon^{\prime 3 / 5}\right)\right)^{2 / 3}, \quad \mathrm{R}_{\mathrm{s}} / \mathrm{r}_{0}=\left(\mathrm{R}_{0} / \mathrm{r}_{0}\right) \cdot\left(\left(1.058+\varepsilon^{\prime}\right) / 1.067\right)^{5 / 3} \tag{8}
\end{equation*}
$$

A solid curve in Figure 2 presents the dependence of $\Delta / r_{0}$ on $M_{s}$ according to Eq. 8. Filled squares in this figure are the results of the HD calculations of Spreiter \& Stahara (1995) recently extended to $\mathrm{M}_{\mathrm{s}}$ smaller than in the original


Fig. 2. Comparison of the HD simulation results with empirical Eq. 8. Spreiter et al. (1966) paper. Previous empirical Eq. 2 (Spreiter et al., 1966) underestimates $\Delta$ (long dashes) while Eq. 3 (Farris \& Russell, 1994) overestimates $\Delta$ (short dashes) for small $\mathrm{M}_{5}$. The dependence of $\Delta / r_{0}$ on $M_{a}$ according to Eq. 8 is shown in Figure 3 by solid and dashed curves for $\vartheta_{\text {bn }}=90^{\circ}$ and $45^{\circ}$, respectively. Here Eq. 5 instead of Eq. 1 was used for the calculation of $\varepsilon$ and $\varepsilon^{\prime}$. Filled circles and squares correspond to results of the MHD simulations of Cairns \& Lyon (1995) for $\vartheta_{\mathrm{bn}}=90^{\circ}$ and $45^{\circ}$, respectively. It is obvious that empirical Eq. 8 is in reasonable agreement with the results of MHD simulations, too.

Three parameters: stand off distance $\Delta$, radius of curvature $R_{s}$ in the nose point, and asymptotic downstream slope $\vartheta_{d}=\arcsin \left(1 / \mathrm{M}_{\mathrm{d}}\right)$ are necessary and sufficient for the determination of a hyperbolic curve $x(y)$ which could be considered as a zero proxy to the bow shock surface:

$$
\begin{equation*}
x(y)=r_{0}+\Delta+R_{s}\left(M_{d}^{2}-1\right)-R_{s}\left(M_{d}^{2}-1\right) \sqrt{1+y^{2} /\left[R_{s}^{2}\left(M_{d}^{2}-1\right)\right]} \tag{9}
\end{equation*}
$$

Fig. 3. Comparison of the MHD simulation results with empirical Eq. 8.
where $y$ is the distance from the x-axis and $M_{m s} \leq M_{d} \leq \min \left(M_{a}, M_{s}\right), M_{m s}$ is magnetosonic Mach number. Two linear over $y$ terms added to the zero proxy curve (9) inside and outside of the square root do not spoil all its useful properties if all coefficients will be properly selected. Above procedure leads to the appearence of single 'shape parameter' $\chi$ which provides possibility to fit the results of HD calculations:

$$
\begin{equation*}
x(y)=r_{0}+\Delta+\chi R_{s}\left(M_{d}^{2}-1\right)-y \sqrt{(1-\chi)\left(M_{d}^{2}-1\right)}-\chi R_{s}\left(M_{d}^{2}-1\right) \sqrt{1-\frac{2 y}{\chi R_{s}} \sqrt{\frac{1-\chi}{M_{d}^{2}-1}}+\frac{y^{2}}{\chi^{2} R_{s}^{2}\left(M_{d}^{2}-1\right)}} \tag{10}
\end{equation*}
$$



Fig. 4. Comparison of the HD calculated bow shocks. Present paper calculations are marked by different symbols. Filled symbols and solid curves correspond to $\gamma=5 / 3$, empty triangles and dashed curves to $\gamma=2$. In (a) for every $\gamma$ three shocks are presented for $M_{s}=2,4$, and 8 , in (b) $M_{s}=8$.

Figure 4 demonstrates the correspondence of the bow shock shapes calculated using Eq. 10 to shapes resulted from the HD modeling for an obstacle of fixed shape and different $\gamma, \mathrm{M}_{\mathrm{s}}$ (Spreiter and Stahara, 1995) (a), and for an obstacle of variable shape (Spreiter et al., 1970) (b). In our calculations we used

$$
\begin{equation*}
\chi=0.5+0.38 \mathrm{R}_{0} / \mathrm{r}_{0}-2.5 \times 10^{-5} \gamma^{5}(\gamma-1)^{3} / \varepsilon^{6} \tag{11}
\end{equation*}
$$

and all other external flow parameters coincident with those used in the HD modeling. (Note possible mislabeling of the second and third shocks from the obstacle in original Figure 12 of Spreiter and Stahara (1995) corrected in Figure 4a).

## APPLICATION TO THE MARTIAN BOW SHOCK OBSERVATIONS

The values of proton density $n_{p}$, velocity $V$, and temperature $T_{p}$ measured by the TAUS ion spectrometer and magnetic field $\overrightarrow{\mathrm{B}}$ measured by the MAGMA magnetometer onboard the Phobos 2 orbiter were used to evaluate $\rho V^{2}$, $\mathbf{M}_{s}, \mathbf{M}_{\mathrm{a}}$ in a manner described by Verigin et al. (1996). Uncertainty caused by the unknown phase of the Phobos 2 rotation around the axis approximately pointing to the Sun is not important for $\vartheta_{\text {bn }}$ determination in the subsolar region of the bow shock. The influence of this uncertainty on the $\mathrm{M}_{\mathrm{d}}$ value selection results in the scatter of the expected bow shock crossings with Phobos 2 orbit, which is much less than the scatter of the observed crossings. We used $M_{d}=M_{m s}$ for certainty. Upstream values of $\rho V^{2}$ were used to determine subsolar distance $r_{0}$ and curvature radius $\mathrm{R}_{0}$ of the magnetopause according to the model of Verigin et al. (1996) with planetary magnetic moment of $0.82 \times 10^{22} \mathrm{G} \mathrm{cm}^{3}$. Thus, values of $\rho \mathrm{V}^{2}, \mathrm{M}_{\mathrm{s}}, \mathrm{M}_{\mathrm{a}}$, and $母_{\mathrm{bn}}$ were used for the calculation of the bow shock surface by Eq. 10 and for the calculation of the expected positions (zenith angle) of the Martian shock crossings in circular


Fig. 5. Comparison of bow shock positions observed at Phobos 2 circular orbits ( -6150 km above Martian surface) with positions calculated for the original (a) and modified (b) values of $\rho V^{2}$. Lines correspond to coincidence in the positions. Phobos 2 orbits.

Figure 5a presents the comparison of the expected (predicted) positions of the bow shock for $\gamma=2$ with those observed onboard Phobos 2 (the use of $\gamma=5 / 3$ results in about $5^{\circ}$ higher zenith angles). General agreement of the calculated and observed bow shock positions seems to be reasonable, though the scatter of the first is less than the scatter of the second. This is possibly connected with temporal variations in the solar wind. Really, the upstream solar wind parameters were measured about half an hour before or after the bow shock crossing and, hence, about one hour and a half before or after the Phobos 2 orbiter crossed the magnetopause. On the other hand, observed magnetopause position can provide information on $\rho V^{2}$ (with the use of the magnetopause model) during the crossing time. Thus determined $\rho V^{2}$ (and properly corrected $\mathbf{M}_{\mathbf{s}}, \mathbf{M}_{\mathbf{a}}$ ) were used for the calculation of predicted bow shock positions presented in Figure 5 b . Now the scatters of the calculated and observed bow shock positions are approximately equal, justifying the influence of the solar wind temporal variations on the scatter of the calculated bow shock crossings.

solar wind ram pressure, dyn/ $\mathrm{cm}^{2}$
Fig. 6. Comparison of the observed and calculated dependencies of the Martian bow shock on $\rho V^{2}$.


Fig. 7. Comparison of the observed and calculated dependencies of the Martian bow shock on $\rho V^{2}$ for two different subsets of data.

Figure 6 presents the dependence of the Martian bow shock positions on the observed upstream $\rho V^{2}$. Solid and dashed curves in this figure were calculated using Eq. 10 for $M_{s}=M_{a}=10, \vartheta_{b n}=0^{\circ}$, and for $M_{s}=M_{a}=4, \vartheta_{b n}=90^{\circ}$, respectively. Both curves show weak dependence of the shock position on the solar wind ram pressure in accordance with observations, thus supporting the reasonability of the Martian bow shock model developed in the present paper.

Two subsets of the Phobos 2 bow shock crossings are shown in Figure 7 by filled circles ( $4<\mathrm{M}_{\mathrm{s}}<6,5<\mathrm{M}_{\mathrm{a}}<8$ ) and by crosses ( $6<\mathrm{M}_{\mathrm{s}}<10,8<\mathrm{M}_{\mathrm{a}}<12$ ). Solid and dashed curves in this figure are calculations for $M_{s}=5, M_{a}=6.5$ ( $\vartheta_{\text {bn }}=0^{\circ}$ ) and for $M_{s}=8, M_{a}=10\left(\vartheta_{b n}=90^{\circ}\right)$, respectively. Again the lines reasonably describe the results of observations.

## CONCLUSIONS

- The quantitative model of the planetary bow shock presented above reasonably well describes its motion and shape variation due to changes in the external plasma flow parameters $\left(\rho V^{2}, M_{s}, M_{a}, \vartheta_{b n}\right)$ and variations of the magnetopause shape $\left(\mathrm{R}_{0}, \mathrm{r}_{0}\right)$.
- Application of this model to the analysis of the Martian bow shock motion permitted to explain some of its unusual properties.
- The model can be applied to the analysis of other planetary bow shocks including shocks with unusually low Mach numbers (see, e.g., Russell \& Zhang (1992)).
- The limits of applicability of the present model need to be studied in more details especially for the MHD flows.


## ACKNOWLEDGMENTS

The authors are grateful to Dr. M.G. Lebedev from Moscow State University for useful discussion of HD aspects of the problem. The research described in this publication was made possible in part by grants $94-982$ from INTAS, $95-02-04223$ from RFFE; OTKA T 015866 from HSF; and by the Hungarian-Russian Intergovernmental S\&T Cooperation Program (Project 28).

## REFERENCES

Belotserkovsky, O.M., A. Bulekbaev, M.M. Golomazov, V.G. Grudnitsky, V.K. Dushin et al., Flow-around of blunt bodies by supersonic gas (theoretical and experimental studies), pp. 1-400, ed. by O.M. Belotserkovsky, AN SSSR Computer Center publ., Moscow, (1967).
Biermann, L., B. Brosowski, and H.U. Schmidt, Solar Phys., 1, 254, (1967).
Cairns, I.H., and J.G. Lyon, J. Geophys. Res., 100, 17173, (1995).
Farris, M.H., and C.T. Russell, J. Geophys. Res., 99, 17681, (1994).
Minailos, A.V., Izvestiya AN SSSR, Mechanics of fluid and gas (in Russian), No.3, 176, (1973).
Russell, C.T. and T.-L. Zhang, Geophys. Res. Letters, 19, 833, (1992).
Shugaev F.V., Applied mathematics and mechanics (in Russian), No.1, 184, (1964).
Schwingenschuh, K., W. Riedler, T.-L. Zhang, H, Lichtenegger et al., Adv. Space Res., 12, (9)213, (1992).
Spreiter, J.R., A.L. Summers, and A.Y. Alksne, Planet. Space Sci., 24, 223, (1966).
Spreiter, J.R., A.L. Summers, and A.W. Rizzi, Planet. Space Sci., 18, 1281, (1970).
Spreiter, J.R., and S.S. Stahara, Adv. Space. Res., 15, (8/9)433, (1995).
Stulov, V.P, Izvestiya AN SSSR, Mechanics of fluid and gas (in Russian), No.4, 142, (1969).
Tatrallyay, M., C.T. Russell, J.D. Mihalov, and A. Barnes, J. Geophys. Res., 88, 5613, (1983).
Verigin, M.I., K.I. Gringauz, G.A. Kotova, A.P. Remizov, N.M. Shutte et al., J. Geophys. Res., 98, 1303, (1993).
Verigin, M., G. Kotova, N.Shutte, A.P. Remizov, K. Szegõ et al., J. Geophys. Res., 102, in press, (1997).
Zhang, T.-L., K. Schwingenschuh, C.T. Russell, and J.G. Luhmann, Geophys. Res. Lett., 18, 127, (1991).
Zhuang, H.C., and C.T. Russell, J. Geophys. Res., 86, 2191, (1981).

