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METHODOLOGY OF EVALUATION OF THE DATA MEASURED BY RETARDING POTENTIAL ANALYZERS

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The methodology of the evaluation of the characteristic curves measured by plane-gridded retarding potential analyzers (planar ion trap) is described. In course of the evaluation first a primary processing of the data is necessary for the conversion of the telemetric data into characteristic curves suitable for the curve fitting procedure. Then the determination of the plasma parameters follows by comparing the experimental characteristic curves with theoretical curves, while the values of the plasma parameters are changed. Finally, the validity of the planar ion trap approximation is discussed.

Keywords: retarding potential analyzer; curve fitting procedure; planar trap approximation

Introduction

Within the scope of the Intercosmos programme retarding potential analyzers have also been used on rockets for the determination of the plasma parameters (Apáthy et al. 1981, Bencze et al. 1985). Thus, the elaboration of a procedure was necessary by means of which plasma parameters can be determined from the telemetric data received at the ground. The procedure consists of two steps. The first step called primary processing includes the conversion of the telemetric data forming a characteristic curve into currents and voltages cutting out those parts of the characteristic curve, where the sensitivity was changed and smoothing the curve by using sliding averages instead of single values. The second step includes the actual determination of the plasma parameters by means of curve fitting (Whipple 1959, Knudsen 1966, Moss and Hyman 1968, Hanson et al. 1970). Since the planar ion traps were used to a height of 1500 km, the consideration of the validity of the planar approximation was also necessary.

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Primary processing of the data

As it is known, in case of a retarding potential analyzer the result of the measurements is a characteristic curve, which represents the current due to positive ions as a function of the retarding potential, i.e. as a function of the energy and mass of the particles. This positive ion current can be expressed as a function of the retarding potential in analytical form. The unknowns in this equation are determined by means of the fitting of the computed characteristic curve to the measured one changing the values of the unknowns. The unknown parameters which determine the dependence of the current on the retarding potential are the plasma parameters, i.e. the concentrations of the different ions, the ion temperature and the rocket (container) potential. The accurate determination of these parameters is possible only in that case, when the scatter of the measured data is small. The analysis of the noise has shown that the noise is produced outside the instrument. Namely, the noise level is independent of the amplification. For the determination of the source of the noise that part of the characteristic curve belonging to large retarding potentials has been analysed, where the signal to noise ratio is small (Fig. 1). First the deviations of the current values from a parabola approximating the data have been computed. The harmonic analysis of these deviations has shown that in the spectrum there are two peaks with periods of 0.012 and 0.007 s, respectively (Fig. 2). The first peak corresponds to the rotation period of the commutator, while the second peak to that of its first harmonic. Thus, according to this analysis the noise originates from the commutator. In the course of the data processing no filtering was used. Practice has shown that the manipulation of the data may lead to incorrect results. Since there are enough experimental characteristic curves, the best curves have been selected on the basis of the scatter of the data.

The retarding potential corresponding to the measured current (during one revolution of the commutator the current is measured four times, but the retarding

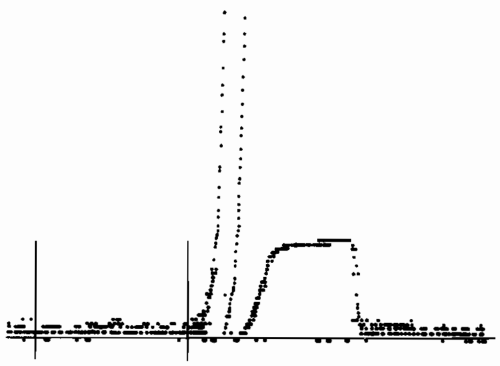

Fig. 1. Part of characteristic (current-voltage) curve which has been used for the determination of the noise spectrum

potential only once) is determined as follows: The retarding potential corresponding to the measured current consists of two parts — one part is connected with the change of the retarding potential during one complete revolution of the commutator, the other part corresponds to its variation between two measurements within one period of revolution of the commutator. The difference between the voltages corresponding to the beginning and end of the characteristic curve is divided by the number of rotations of the commutator (the period of the saw-tooth voltage divided by the period of the commutator). Thus, the change of the retarding potential during one revolution of the commutator is obtained. Then dividing the change of the retarding potential during one revolution of the commutator by the number of channels of the commutator the change of the potential between measurements carried out in adjoining channels is found. Since the current was measured in every eighth channel, the retarding potential was calculated by means of the following formula:

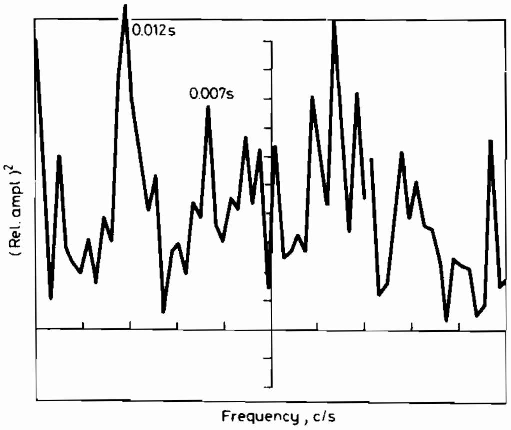

Fig. 2. The noise spectrum of the characteristic curve shown in Fig. 1

$$V=16-(I-1)C_k-8JK_k\,,$$

where C_k is the change of the retarding potential during one revolution of the commutator, K_k being the variation of the potential between measurements carried out in two adjoining channels (in case of the measurement of the current every eighth channel is used), $I=1, \ldots N$ and $J=0, 3$.

Preparing the current data for the computation of the parameters those parts of the experimental characteristic curves have been omitted, where the amplification respectively the sensitivity of the equipment was changed. In order to reduce the scatter

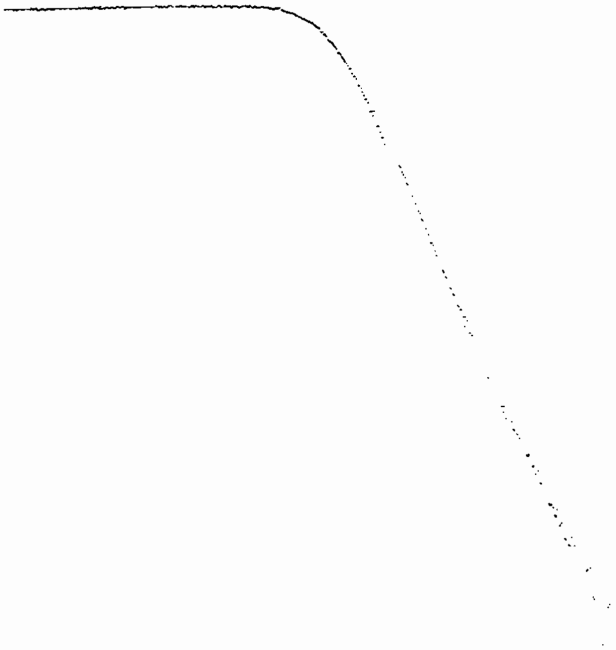

Fig. 3a. Characteristic curve before smoothing

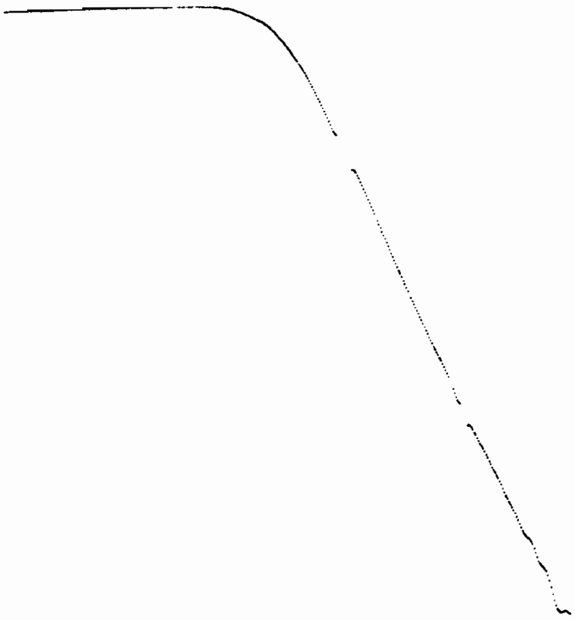

Fig. 3b. The same curve after smoothing

of the data in different parts of the curve recorded with the same sensitivity the data were smoothed using the formula

$$y_n = \frac{S_1 y_{n-1} + S_2 y_n + S_3 y_{n+1}}{S_1 + S_2 + S_3}$$

where $S_1 = S_3 = 1$ and $S_2 = 2$ are weighting coefficients. This procedure is repeated ten times. The efficiency of the smoothing is illustrated by Figs 3a and b, where the same experimental characteristic curve is shown respectively before and after smoothing. Before starting the data processing the telemetric data are converted into current intensities. This conversion is performed by means of the data obtained by checking the amplifier in different ranges of amplification. The current intensity is calculated from the formula

$$i = (TD - 0\%) \frac{6300}{100\% - 0\%} \cdot G$$

where TD is the current in telemetric units, 0% and 100% are respectively the beginning and end of the measuring range in telemetric units, 6300 is the calibrating voltage in millivolts, G being the sensitivity of the amplifier corresponding to the given range.

Determination of the plasma parameters

The parameters of the plasma are determined by fitting the computed curve (on the basis of the equation of the characteristic curve) to the measured characteristic curve, while the values of the unknowns, i.e. the magnitude of the plasma parameters are changed. The curve fitting is started with approximate values of the unknown plasma parameters. As it is known, the relation of the ion current i to the retarding potential V is given by the equation (Whipple 1959, Knudsen 1966):

$$i = \alpha A e \sum_j n_{ij} \left\{ v_r \cos \Theta \left[\frac{1}{2} - \frac{\operatorname{erf}(X_j)}{2} \right] + \frac{C_{ij} \exp(-X_j^2)}{2\sqrt{\pi}} \right\};$$

$$X_j = \frac{1}{C_{ij}} \left[\sqrt{\frac{2e(v+\varphi)}{m_{ij}}} - v_r \cos \Theta \right]$$

where α is the transparency of the grids, A is the area of the aperture of the trap, v_r is the velocity of the container, n_{ij} and m_{ij} are respectively the concentration and the mass of the positive ions of type j, $C_{ij} = \sqrt{\frac{2kT_i}{m_{ij}}}$ being the most probable velocity of the j-th ion, T_i is the common ion temperature, Θ is the angle between the direction of the relative velocity vector and the normal to the trap aperture (angle of attack), φ is the potential of the container relative to the undisturbed plasma.

The values of the plasma parameters are obtained, when the sum of the deviations of the computed current intensities (forming the theoretical characteristic curve) from the measured current intensities becomes minimum. Since the number of measurements (the number of related voltages and current intensities), determining a characteristic curve is greater than the number of unknowns, the problem is overdetermined and can be solved by means of the mathematical statistics. Expanding in series the equation above the following expressions are obtained:

$$v_1 = a_1 \delta x_1 + b_1 \delta x_2 + c_1 \delta x_3 + d_1 \delta x_4 + e_1 \delta x_5 - l_1$$

$$v_2 = a_2 \delta x_1 + b_2 \delta x_2 + c_2 \delta x_3 + d_2 \delta x_4 + e_2 \delta x_5 - l_2$$

$$\vdots$$

$$v_n = a_n \delta x_1 + b_n \delta x_2 + c_n \delta x_3 + d_n \delta x_4 + e_n \delta x_5 - l_n$$

where

$$a = \alpha A e \left\{ v_r \cos \Theta \left[\frac{1}{2} - \frac{\operatorname{erf}(X_{O^+})}{2} \right] + \frac{C_{O^+} \exp(-X_{O^+}^2)}{2\sqrt{\pi}} \right\} \qquad \delta x_1 = \delta n_{O^+}$$

$$b = \alpha A e \left\{ v_r \cos \Theta \left[\frac{1}{2} - \frac{\operatorname{erf}(X_{He^+})}{2} \right] + \frac{C_{He^+} \exp(-X_{He^+}^2)}{2\sqrt{\pi}} \right\} \qquad \delta x_2 = \delta n_{He^+}$$

$$c = \alpha A e \left\{ v_r \cos \Theta \left[\frac{1}{2} - \frac{\operatorname{erf}(X_{H^+})}{2} \right] + \frac{C_{H^+} \exp(-X_{H^+}^2)}{2\sqrt{\pi}} \right\} \qquad \delta x_3 = \delta n_{H^+}$$

$$d = \alpha A e \sum_j n_{ij} \left\{ v_r \cos \Theta \frac{1}{2} \frac{x_j \exp(-X_j^2)}{\sqrt{\pi}\, T} + \frac{\exp(-X_j^2)}{2\sqrt{\pi}} \left(\frac{k}{mc} + \frac{c_{ij} X_j^2}{T} \right) \right\} \qquad \delta x_4 = \delta T_i$$

$$e = \alpha A e \sum_j n_{ij} \left\{ -v_r \cos \Theta \frac{1}{\sqrt{\pi}} \frac{\exp(-X_j^2) e}{C_{ij} \sqrt{\frac{2e(v+\varphi)}{m_{ij}}}\, m_{ij}} - \frac{\exp(-X_j^2)}{\sqrt{\pi}} X_j \frac{e}{\sqrt{\frac{2e(v+\varphi)}{m_{ij}}}\, m_{ij}} \right\}$$

$$\delta x_5 = \delta \varphi$$

$$l = \alpha A e \sum_j n_{ij} \left\{ v_r \cos \Theta \left[\frac{1}{2} - \frac{\operatorname{erf}(X_j)}{2} \right] + \frac{C_{ij} \exp(-X_j^2)}{2\sqrt{\pi}} \right\} - i$$

and the coefficients of the observation equations are computed by means of the approximate values of the unknowns. The same equations in matrix form are

$$v = \mathbf{A}x - y\,.$$

Since the condition

$$\sum_{i=1}^{n} v_i^2 = \mathrm{Min}$$

is equivalent to the condition

$$\sum_{i=1}^{n} v_i \frac{\partial v_i}{\partial x_j} = 0 \qquad (j=1,2, \dots k)$$

from equation

$$\frac{\partial v'}{\partial x_j} \cdot v = e'_j \mathbf{A}'(\mathbf{A}x - y) = e'_j(\mathbf{A}'\mathbf{A}x - \mathbf{A}'y) = 0$$

the formula for the computation of the unknowns is obtained as

$$x = \frac{\mathbf{A}'y}{\mathbf{A}'\mathbf{A}} = \mathbf{A}'y(\mathbf{A}'\mathbf{A})^{-1},$$

where $\mathbf{A}'$ is the matrix transpose.

This procedure is repeated till the supplementary quantity δx_5 (supplementary quantity for the common ion temperature) will be less than about 1% of the approximate value. The characteristic curves included in the further analysis are selected on the basis of the mean square error. This quantity is determined by the deviation of the measured from the computed curves on the basis of the parameters of the characteristic curves.

On the validity of the planar ion trap approximation

As it is known, the rockets type Vertical reach an altitude of 1500 km. Since the density of the plasma decreases with increasing height, the mean free path of the ions increases and becomes commensurable with the dimensions of the trap. However, the planar ion trap approximation used here presumes that the flux density of the positive ions is constant over the surface of the aperture and the plasma sheath, as well as the electric field of the grids affect only the velocity component of the ions perpendicular to the aperture. These conditions are fulfilled only in case of small Debye length, that is in the dense layers of the ionosphere. Computations show that in case of Debye lengths less than 10 cm, the retarded positive ion current can be determined on the basis of the planar approximation with an accuracy better than 10%, if the angle of attack is <45°, the potential of the aperture grid is equal to the potential of the body (container) (no drawing-in potential) and the negative potential of the body is less than 3 V (Parker and Whipple 1970). The planar approximation does not give accurate results in case of large Debye lengths, if the retarding potential is small. Since the Debye-length does not reach 10 cm in 1500 km altitude, the planar trap approximation gives reliable results to these heights. However, the deviation of the circumstances from the above mentioned conditions may result in scatter of the data.

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