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ON THE INFLUENCE OF THE IONOSPHERE ON THE
RADIOTRACKING OF SPACE ROCKETS

G. L. Gdalevich, K. I. Gringauz, V. A. Rudakov, and S. M. Rytov
U. S. S. R. Academy of Sciences, Moscow, U. S. S. R.

(With 6 Figures)

Abstract

The report contains some considerations relating to the possibility of calculating the errors caused by the ionosphere during radiotracking, i. e., during the determination of space vehicle coordinates and velocities by radio methods. Expressions are given for these errors depending on the ionosphere parameters. The possibility of the approximation of the actual altitude electron density distributions for the estimation of the errors under consideration is discussed.

A propos de l'influence de l'ionosphère sur la détermination de position des fusées cosmiques. L'exposé contient quelques considérations, concernant la possibilité de calcul des fautes ionosphériques dans la détermination des coordonnées et de la vitesse des fusées cosmiques par méthodes radioélectriques. Les expressions de ces fautes sont présentées sous la dépendance des paramètres ionosphériques. On discute la possibilité d'une approximation de la distribution par hauteur réelle de la concentration électronique pour le calcul des fautes considérées.

К вопросу о влиянии ионосферы на определение положения космических ракет. Доклад содержит некоторые соображения, относящиеся к возможности вычисления погрешностей, вносимых ионосферой при определении радиометодами координат и скорости космических ракет. Приводятся выражения для этих погрешностей в зависимости от параметров ионосферы и обсуждается возможность аппроксимации реального высотного распределения электронной концентрации для целей вычисления рассматриваемых погрешностей.

I. Introduction

At the XIth International Astronautical Congress in Stockholm in the report of Mullen and Woods, it was pointed out that the errors caused by the ionosphere in the determination of space vehicle coordinates by radio

methods can be minimized by means of one of the following methods :

- 1) the use of very high frequencies;
- 2) the simultaneous reception of several frequencies;
- 3) the introduction of corrections which take into account the state of the ionosphere ¹.

Although the first two methods are evidently preferable, the treatment of the problems connected with the errors (introduced into the radio measurements of the coordinates and velocities of space vehicles) and with the possibilities of the corresponding corrections, deserves attention for the following reasons. With the increase of the accuracy of the instrumentation the relative weight of the propagation errors will increase. Besides, it is not excluded that astronautics will deal with measurements from some planet whose ionosphere is characterized by considerably higher electron densities than the Earth's and at which the use of sufficiently high (for terrestrial conditions) radio frequencies will not make the errors in the determination of the coordinates small enough.

The interest in the questions connected with ionospheric errors is confirmed by the recent publication of several articles, for instance, by Millman in 1958 ², Weisbrod and Anderson in 1959 ³, Kelso in 1960 ⁴, and Lowen in 1962 ⁵.

An exhaustive treatment of the problem of ionospheric errors in the measurements of the coordinates and velocity of space rockets by radio methods should not only include the theory of these errors which takes into account the influence of inhomogeneities of the electron density n_e and its time variations, but also a complete survey of the data available on the structure of the ionosphere and its changes, a consideration of different methods of analytic approximations of real spatial distributions of n_e , an estimate of the accuracy of such approximations, and the possibilities of the rapid determination of the state of the ionosphere in its entire height, and, last, the methods of the use of the results of such determination for the calculation of ionospheric errors.

The present report does not give such a complete treatment. Only some details of the problem are considered.

Giving expressions to the errors of radio measurements of the coordinates and radial velocity, we shall apply them to the case of a space rocket which is beyond the ionosphere. Errors introduced by the troposphere and interplanetary plasma are not taken into account.

Because of the necessity of the rapid determination of the actual height distribution of the electron density $n_e(h)$, data are given on the comparison of simultaneously obtained results of rocket measurements and measurements made by means of the ionospheric station.

II. Expressions for Ionospheric Errors of Radiomeasurements of the Range, Elevation, and Velocity of Space Rockets

Let us introduce a spherical coordinate system with the origin at the point of observation (Fig.1). The coordinates of the point of the ray A will be r, θ, φ . The elevation $\beta = \pi/2 - \theta$, and the range R shows the

actual position of the required to find :

- a) the refraction in t

$$\Delta \beta = ($$

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- b) the error in the r

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- c) The error in the r

$$\Delta R_{imp} =$$

where c is the light velocity, f is the frequency

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quired to find :

a) the refraction in the horizontal and vertical planes, namely

$$\Delta \beta = (\beta^{\text{meas.}} - \beta) r = 0, \quad \Delta \varphi = \varphi^{\text{meas}}$$

where $\beta^{\text{meas.}}$ is the measured elevation and β is the actual elevation and
where $\varphi^{\text{meas.}}$ is the measured azimuth;

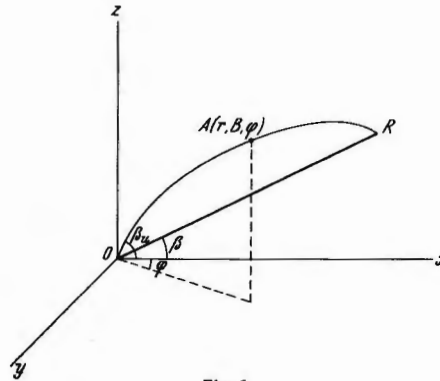


Fig. 1

b) the error in the range determined by the phase method

$$\Delta R_{\text{phas}} = R_{\text{phas}} - R = \int_0^R n dr - R,$$

where the integration is performed along the ray, n is the refraction
index;

c) The error in the range determined by the impulse method

$$\Delta R_{\text{imp.}} = R_{\text{imp.}} - R = \int_0^R \frac{c}{v_{gr}} dr - R = \int_0^R \frac{dr}{\frac{\partial f}{\partial (nf)}} - R,$$

where c is the light velocity in vacuo, v_{gr} is the radio wave group veloc-
ity, f is the frequency of the oscillations used.

The consideration is performed to an approximation of geometric optics
for radio waves with frequencies $f \geq 5 \cdot 10^7$ cps, at which the square of the
refraction index can be expressed with sufficient accuracy as

$$n^2 = 1 - 8.08 \cdot 10^7 \frac{n_e}{f^2} \quad (1)$$

where n_e is the electron density (per cm^3), f is frequency (in cps).

Since in the considered wavelength range the value $(1-n^2)$ does not exceed 0.1 at the maximum observed electron densities in the ionosphere, it is possible to transit from general refraction formulas in which integration is performed along the ray to the perturbation method, taking for a zero approximation the ray trajectory in a homogeneous medium with the refraction index $n=1$, i. e., the actual direction to the rocket.

Taking into account that angles $\Delta\beta$ and $\Delta\varphi$ are not lower than the first order of magnitude with respect to $1 - n^2$, one can write the expansion

$$n^2 = 1 + \alpha (r, \theta, \varphi) = 1 + \alpha_0 + \gamma \cdot \Delta\beta + \delta \cdot \Delta\varphi + \epsilon \cdot \Delta\beta^2 + \xi \cdot \Delta\beta \cdot \Delta\varphi + \eta \cdot \Delta\varphi^2 + \dots,$$

where

$$\alpha_0 = \alpha (r, \theta, 0); \quad \gamma = \left[\frac{\partial (1-n^2)}{\partial \theta} \right]_0; \quad \delta = \left[\frac{\partial (1-n^2)}{\partial \varphi} \right]_0;$$

$$\epsilon = \frac{1}{2} \left[\frac{\partial^2 (1-n^2)}{\partial \theta^2} \right]_0; \quad \xi = \left[\frac{\partial^2 (1-n^2)}{\partial \theta \cdot \partial \varphi} \right]_0; \quad \eta = \frac{1}{2} \left[\frac{\partial^2 (1-n^2)}{\partial \varphi^2} \right]_0.$$

The index "0" means that the corresponding quantity is considered as a function of r on the actual direction. Integrating the ray differential equation, let us find the ray trajectory

$$\frac{dr}{\partial \psi} = \frac{r}{r} \frac{d\theta}{\partial \psi} = \frac{r \cdot \sin \theta}{r \sin \theta} \cdot \frac{d\varphi}{\partial \psi} \tag{2}$$

in which $\psi (r, \theta, \varphi)$ characterize the wave phase, and the function $\psi (r, \theta, \varphi)$ is a solution of the eiconal equation

$$(\nabla \psi)^2 = n^2. \tag{3}$$

The solution can be taken in the form of a series in powers $1 - n^2$ bearing in mind that to a zero approximation $\psi_0 = r$:

$$\psi (r, \theta, \varphi) = r + \psi_1 (r, \theta, \varphi) + \psi_2 (r, \theta, \varphi) + \dots$$

It should be noted that

$$\psi_m (0, \theta, \varphi) = 0, (m = 1, 2, \dots).$$

By substituting the expansions for n^2 and $\psi (r, \theta, \varphi)$ into Eq. (3) we obtain equations of successive approximations which are integrated without difficulties. As a result we obtain the expression for ψ with the desired

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order of accuracy with respect to $1-n^2$. Then, having determined the trajectory of the ray to a first approximation from Eq. (2), i. e., through ψ_1 , it is possible to substitute again the expressions obtained for $\Delta\beta$ and $\Delta\varphi$ into Eq. (2), but written already with the next order of accuracy, to integrate them once more and thus to obtain corrections of the second order. In such a way it is possible to calculate the values we are interested in with the required accuracy.

Let us cite the results of the first approximation obtained for the values we are interested in.

a) The expression for the ionospheric error of the measurement of the range is

$$\Delta R_{\text{phas}} = -\Delta R_{\text{imp}} = -\frac{4.04 \cdot 10^7}{f^2} \int_0^R n_e dr, \tag{4}$$

where f is the frequency (in cps), $n_e = n_e(r, \pi/2 - \beta, 0, t)$ is the concentration of free electrons per cm^3 .

b) The expression for the ionospheric error of the measurement of the elevation is

$$\Delta\beta = \frac{4.04 \cdot 10^7}{f^2} \int_0^R \left(\frac{1}{r} - \frac{1}{R} \right) \frac{\partial n_e}{\partial \beta} dr. \tag{5}$$

c) The expression for the ionospheric error of the measurement of the azimuth is

$$\Delta\varphi = -\frac{2.02 \cdot 10^7}{f^2 \cdot \cos \beta} \int_0^R \left(\frac{1}{r} - \frac{1}{R} \right) \frac{\partial n_e}{\partial \varphi} dr. \tag{6}$$

d) The ionospheric error of the measurement of the radial velocity can be expressed as

$$\Delta \dot{R} = \frac{4.04 \cdot 10^7}{f^2} \left[n_e^1 \dot{R} + \int_0^R \frac{\partial n_e}{\partial t} dr \right] \tag{7}$$

where n_e^1 is the electron density per cm^3 at the point where the rocket is situated, \dot{R} is the radial velocity.

Eqs. (4) to (7) are applicable at any sufficiently smooth spatial distributions of the electron concentration in the ionosphere. Since these equations are derived in the approximation of the geometric optics, they enable us to treat only such inhomogeneities the linear dimensions of which "a" satisfy the condition $a \gg \sqrt{\lambda r}$ (λ is wave length).

This condition is always satisfied if we speak about the average electron density without the local inhomogeneities taken into account.

In many cases this average distribution is considered to be spherically symmetric (especially due to the absence of reliable data on the dimen-

sions of horizontal inhomogeneities and values of horizontal gradients at different heights).

The expression for the errors in radio measurements of the elevation inserted by the ionosphere acquires the form :

$$\Delta \beta = \frac{4.04 \cdot 10^7}{f^2} \int_0^R \left(\frac{1}{r} - \frac{1}{R} \right) \frac{\partial R_0}{\partial \beta} \frac{\partial n_e}{\partial h} dr, \quad (8)$$

where R_0 is the distance from the Earth's center to the vehicle, and h is the height of the rocket above the Earth's surface.

In the case of the rocket which is situated in interplanetary space at such a large distance that its angular velocity can be considered equal to that of the Earth's diurnal rotation, the error of the radial velocity measured using the Doppler Effect according to Eq. (7) is

$$\Delta \dot{R} = \frac{2.02 \cdot 10^7}{f^2} \frac{1+m^2}{m^2} \left[n_e^1 \dot{R} + \rho \dot{\beta} \cos \beta \int_{\rho}^{R_0} \frac{\partial n_e}{\partial h} \left(1 - \frac{\rho \cdot \sin \beta}{\sqrt{R_0^2 - \rho^2 \cos^2 \beta}} \right) dh + \int_0^R \frac{\partial n_e}{\partial t} dr \right] \quad (9)$$

where m is the ratio of the frequency of the interrogation to the frequency of response, ρ is the Earth's radius, $\dot{\beta}$ is the angular velocity of the Earth's rotation.

III. On the Possibilities of the Consideration of Ionospheric Errors

As is evident from Eqs. (4) to (7), in order to introduce corrections (which take into account the ionospheric errors) into the values of the space rocket coordinates and radial velocity measured by radio methods, it is necessary to know the structure of the ionosphere and the processes which take place in it at the moment of measurements. (Consideration of the method of excluding the ionospheric errors based on simultaneous measurements by means of radio waves of several frequencies is beyond the limits of the present report). Therefore, for the determination of ΔR it is necessary to know the integral electron content along the ray passing through the ionosphere from the observer to the rocket. To determine angular corrections, it is necessary to know transversal gradients n_e on the indicated ray and for corrections of the measured radial velocity it is necessary to know also the rate of the change of the integral electron content.

Any attempt to calculate such corrections leads to the necessity of idealizing the actual structure of the ionosphere since there are no

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means by which it is possible to know the instantaneous spatial distribu-
 tion n_e in the entire region of the ionosphere which influences the meas-
 urements. As was indicated above, due to the absence of reliable data
 about horizontal gradients n_e and the dimensions of inhomogeneities, one
 usually uses the supposition about the spherical symmetry of the iono-
 sphere. The main difficulty is the determination of the vertical distribu-
 tion of n_e . It is known that in the ionosphere, n_e varies considerably in
 time. In order to illustrate the dependence of the vertical distribution of
 n_e on the phase of the cycle of solar activity, one can use curves $n_e(h)$
 given in Fig. 2. Both distributions were obtained by the method of the USW

dispersion interferometer (Gringauz and Rudakov⁶) dur-
 ing the vertical launchings of the geophysical rockets of the U. S.
 S. R. Academy of Sciences over the same geographic point, in
 the same season (autumn), and at the same time of day
 (at daytime). Curve 1 was obtained on October 31, 1958, at
 15h 54 m and curve 2 on November 15, 1961, at 15 h 00 m.

It would be convenient to calcu-
 late the considered errors if
 the altitude variation n_e in the
 ionosphere were always descri-
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 analytic expression whose pa-
 rameters were determined from
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 of the ionospheric stations).

However, our attempts to
 describe in such a way the alti-
 tude distributions n_e obtained
 experimentally by different
 authors during the last five
 years and published in articles by Gringauz^{7,12}, Alperi et.al.⁸, Berning⁹,
 Nisbet¹⁰, Jackson and Bauer¹¹, Pope¹³, Bowles¹⁴, i. e., to find the
 general analytical approximation for them, failed. Among the laws of the
 changes of n_e with height, by means of which we attempted to approxi-
 mate the above indicated experimental results, was the combination of the
 parabola (for the part of the ionosphere lying below the maximum of the
 F-layer) with the exponent (for the remaining part of the ionosphere).

The possibility that the failure of such attempts to find a general approx-
 imation for all experimental curves is partially due to the fact that the ex-
 periments were carried out by different methods and that only in some

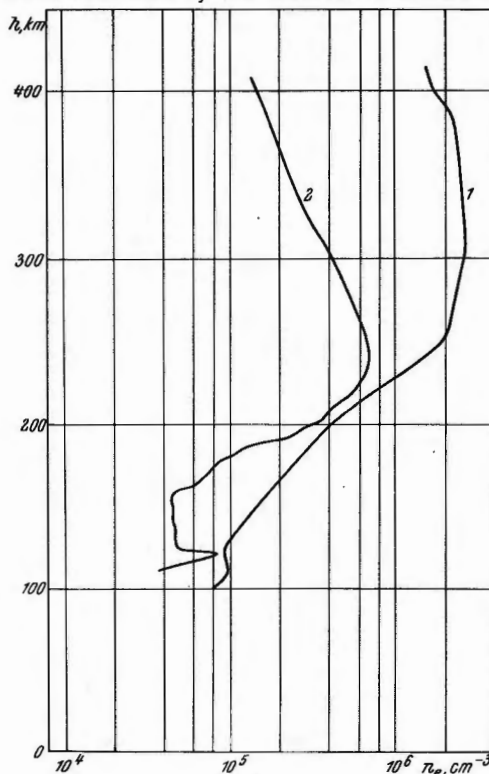


Fig. 2

cases were the errors of the results of measurements properly estimated.

However, one should bear the following in mind. According to the results obtained in recent years, the Earth's ionosphere is definitely stratified and as to the chemical composition of ions, experiments aimed at direct determination of the ion mass-spectrum on Sputnik III (Istomin ¹⁵) have shown that at heights of 300-1,000 km more than 90% of ions are the ions of atomic oxygen. Results to the same effect were obtained by means of ion traps on the same satellite (Gringauz ¹⁶). From the data obtained by means of charged particle traps on the first Soviet space rockets, conclusions were made that at heights of more than 2,000 km (to heights of 15,000-20,000 km) hydrogen ions prevail (Gringauz et al. ^{17, 18}). At last, observations of helium twilight emission (Shefov ¹⁹), as well as considerations based in a considerable degree on the analysis of the observed satellite drag (Nicolet ²⁰) and data of ion traps on a high-altitude rocket Scout ST-7 (Hanson ²¹) and Explorer 8 (Bourdeau et al. ²²), have led to the conclusion that a helium region exists between the ionosphere's oxygen and hydrogen regions.

In a number of these papers ^{11, 12, 17, 21, 22}, the authors, in fact, proceed from the supposition that in each of the ionosphere's region with a homogeneous chemical ion composition, the vertical distribution of the ion density (as well as neutral particle distribution) can be described by a barometric formula.

This supposition is neither apparent nor well-reasoned in any of the papers. For instance, an analysis of the possible difference of the ion height distribution from that of the neutral particles caused by the action of the magnetic field on the ions is nowhere to be found. At the same time this supposition is rather natural. It leads to the conclusion that, beginning with heights $h = 300$ km, the distribution of $n_e(h)$ should be represented by a linear combination of several exponential curves with different power indexes. It is possible that the accumulation of experimental data on the topside ionosphere and the comparison of them with simultaneous data on the lower part of the ionosphere will make it possible in the future to construct the electron concentration distribution correctly throughout the entire height of the ionosphere on the basis of measurements carried out below the maximum of the F-layer.

In this connection, a question of some interest concerns the possibility of the reliable determination of dependencies $n_e(h)$ from the data of the height - frequency characteristics obtained at the ionospheric stations. With this aim Gringauz and Gdalevich ²³ compared $n_e(h)$ distributions calculated from the height - frequency characteristics by the Shinn-Kelso method with $n_e(h)$ dependencies simultaneously obtained by the dispersion method by means of vertically launched geophysical rockets of the U. S. S. R. Academy of Sciences. The ionospheric station was situated near the launching pad. In Fig. 3 results of calculations are shown (by the mark 0 for five points, and by x for ten points) according to the data of height-frequency characteristics obtained during the launching of a rocket on October 31, 1958. The solid curve represents the result of measure -

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properties, in fact, provide a picture of the ionosphere's region with a detailed distribution of parameters (1) can be described. It is mentioned in any of the papers on the difference of the ionosphere caused by the action of the sun. At the same time, it is shown that, beginning from a certain height, it could be represented by a curve with different power-law dependencies. Experimental data on the ionosphere from simultaneous data obtained from rockets are available in the future to be used directly throughout the measurements carried out

concerns the possibility of obtaining $n_e(h)$ from the data of the ionospheric station. The $n_e(h)$ distribution is obtained by the Shinnitskiy method by the physical rockets of the station. The station was situated at a height of 100 km and are shown (by the data of the station) according to the data of the ionosphere. The launching of a rocket is the result of measure -

ments by a dispersion interferometer.

As is evident from Fig. 3, the results of the calculations made from the data of the ionospheric station agree well with the results of direct measurements of $n_e(h)$. Thus it can be considered that at present there is a reliable method of obtaining the actual $n_e(h)$ distribution at heights up to the maximum of the F-layer.

However, before acquisition of a sufficient amount of experimental data on the topside ionosphere, it is impossible to use ionospheric station measurements for the construction of the approximate electron density

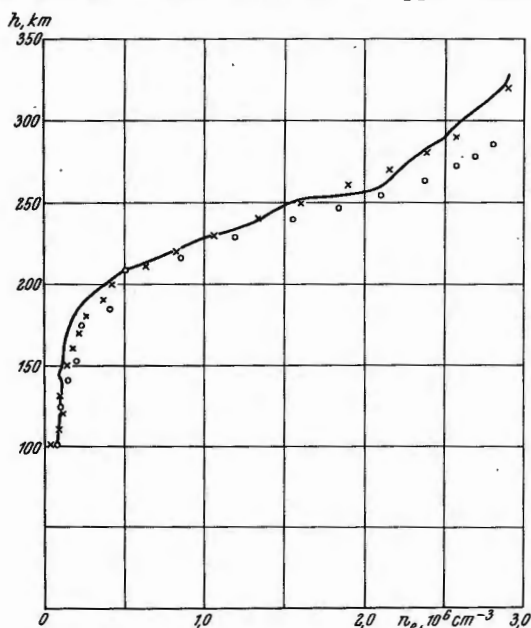


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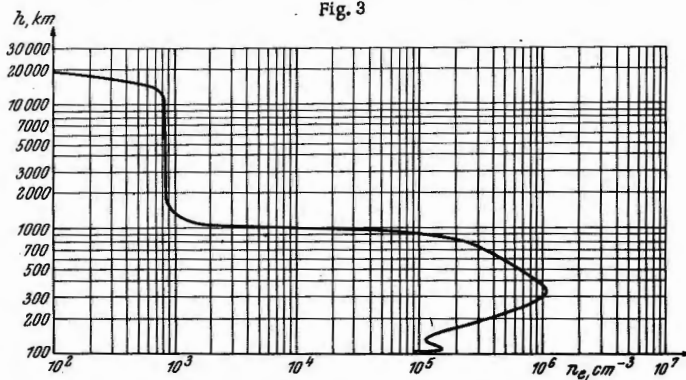


Fig. 4

distribution throughout the entire height of the ionosphere. For the calculation of the considered errors of measurements, it is apparently neces -

sary always to use the actual altitudinal n_e distribution which corresponds to the time of measurements.

Let us give an example of the quantitative estimates of ionospheric errors in the determination of R, β , and \dot{R} of a space rocket by radio methods. The rocket is located beyond the ionosphere, and the wavelength used is $\lambda = 1$ m, the distribution of $n_e(h)$ is shown in Fig. 4. This is a tentative height variation, n_e corresponding to the daytime in the period close to the solar activity maximum based on the data of direct measurements. The results of the calculations made according to Eqs. (4) and (8) are presented in Figs. 5 and 6, respectively. The values of ΔR are

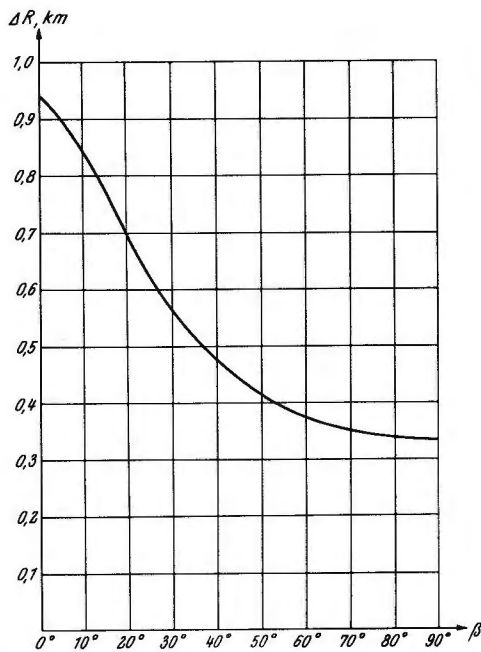


Fig. 5

obtained with the double path of radio waves which are taken into account. at $H = 500$ km amounts to $5 \cdot 10^9$ $\text{cm}^{-2} \text{sec}^{-1}$. If one adopts this magnitude for the value

$$\int_0^R \frac{\partial n_e}{\partial t} dr$$

the error in determining the velocity only according to the time variation of n_e will amount to $\Delta \dot{R} \approx 2 \text{ cm. sec}^{-1}$ as shown in Eq. (9) For the whole thickness of the ionosphere and for small elevations, this estimate should apparently be increased and the last term in Eq. (9) should therefore be taken into account.

It should be noted that, since for the calculation of the value of the ionospheric error in the measurement of the range ΔR , it is necessary to know only the integral electron content, the required data can be obtained from

The estimate of the error of radio measurements of the radial velocity of a space rocket above the ionosphere can be made according to Eq. (9). If we suppose that n_e does not vary in time, i. e., omit the last term in Eq. (9), then at $\lambda = 1$ m, $m = 1$ and, $n_e(h)$ (Fig. 4) we conclude that the error in determining the velocity due to the Earth's rotation amounts to 3.3 cm. sec^{-1} at $\beta = 10^\circ$ and to $0.13 \text{ cm. sec}^{-1}$ at $\beta = 75^\circ$.

It should be noted that according to Ref. 6 (Gringauz, Rudakov) the value

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- 1) Mullen, Congress p. 246.
- 2) Millman,
- 3) Weisbro
- 4) Kelso, J
- 5) Lowen, I
- 6) Gringauz (1961).
- 7) Gringauz
- 8) Alpert, Uspekhi
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measurements of the Faraday effect from radio echo observations of the Moon similar to measurements made for the first time by Evans ²⁴.

From the viewpoint of the calculation of considered ionospheric errors, the most promising is the method of determining $n_e(h)$ based on the use of incoherent scattering of radio waves by free electrons. This method makes possible a rapid determination of $n_e(h)$ throughout the entire thickness of the ionosphere (Gordon ²⁵).

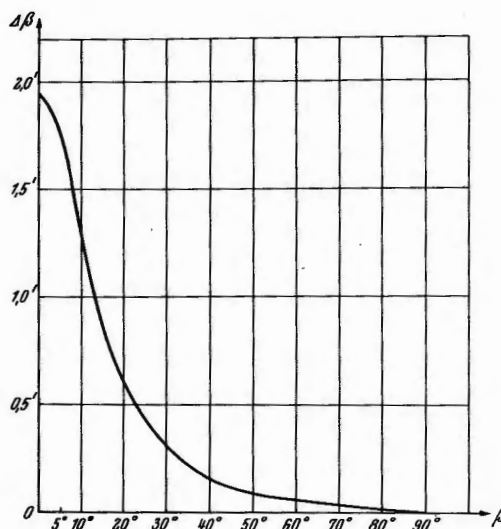


Fig. 6

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