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ANALYSIS OF RESULTS OF SIMULTANEOUS MEASUREMENTS OF ELECTRON CONCENTRATION IN THE IONOSPHERE BY IONOSPHERE STATIONS AND ROCKETS

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All methods for obtaining the height distribution of electron concentration from height-frequency characteristics are based on the integral equation

$$H_0 = \int_0^H \frac{dh}{n}, \tag{1}$$

where H_0 is the "operative" altitude of reflection, H the true height of reflexion and n the refractive index for radio waves with a frequency ω.

There are two basic groups of methods for finding $n_e(h)$, i.e. the height distribution of the electron concentration n_e, from height-frequency characteristics. These methods are those called "comparison (or selection) methods" and methods based on the solution of an integral equation.

When the comparison methods are used a number of distributions $n_e(h)$ expressed analytically are set. When each of these $n_e(h)$ distributions is substituted in (1) the problem is reduced to taking the integral. The experimental height-frequency characteristics are compared with those obtained from integration by a family of height-frequency characteristic curves from which the curve which is closest to the experimental one is selected. The $n_e(h)$ distribution used in the calculation of this characteristic curve, i.e. the closest to the experimental, is also taken as the true distribution.

The second group of methods is connected with the "precise" solution of equation (1). In this case the solution will be simple only if the distribution $n_e(h)$ is a steady function. In actual fact the steadiness of $n_e(h)$ is just as necessary to be able to use the comparison methods (in order to obtain a simple solution).

Since it was mainly thought that the ionosphere had a layered structure (i.e. that the height distribution of n_e was not steady in nature) before rocket research started methods for solving the integral equation were not extensively used at the time despite the fact that they were known as early as 1937(1). There were also practical difficulties in the use of these methods at the time because there were none of the electronic computers necessary for making the calculations which are extremely laborious.

After rocket studies of the ionosphere had shown that the deviations of the function $n_e(h)$ from a steady state are in actual fact, slight interest in these methods increased and they began to be used more widely(2–5).

It should, however, be pointed out that we know only one work(2) up to the present in which a comparison is made of results from determining $n_e(h)$ by the integral equation method from an experimental height-frequency characteristic curve with the $n_e(h)$ distribution up to an altitude of 200 km obtained at the same time with rockets.

It is moreover clear that for a final evaluation of the applicability and accuracy of determining $n_e(h)$ from height-frequency characteristic curves we need many similar comparisons up to the altitude of the chief ionization maximum in the ionosphere if possible. The present article is an attempt partly to fill in this gap.

1. CALCULATION OF HEIGHT-FREQUENCY CHARACTERISTIC CURVES ON BASIS OF $n_e(h)$ DISTRIBUTIONS OBTAINED WITH ROCKETS

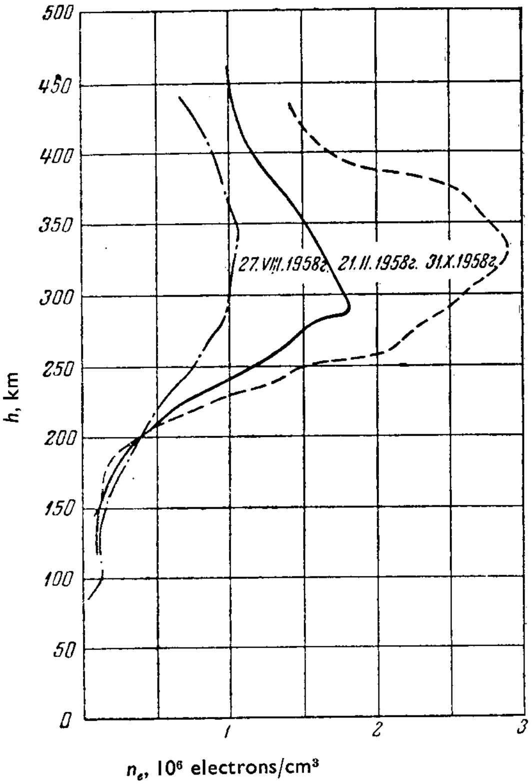

FIG. 1.

Figure 1 gives the results of rocket measurements of $n_e(h)$ made early in the morning and during the day(6). Fig. 1 shows that the parts of the $n_e(h)$ curves below the main ionization maximum deviate only slightly from a steady state (in the form of small maxima). It can, however, be shown that with these height distributions $n_e(h)$ breaks of about 100 km in altitude must be observed in the height-frequency curves obtained by ionosphere stations. (The existence of these breaks at the time helped the formation of the concept of a "layered" ionosphere.) In order to simplify the calculation we shall show it for the case when no account is taken of the dependence of the refraction index n on the magnetic field and the electron collision frequency, i.e. we shall take the expression for n in the form

$$n = \sqrt{1 - \frac{4\pi e^2 n_e(h)}{m\omega^2}}\,. \tag{2}$$

Here e and m are the electron's charge and mass, ω the frequency of the radio wave and $n_e(h)$ the electron concentration at the height h.

By breaking down the electron concentration distribution curve obtained by rocket

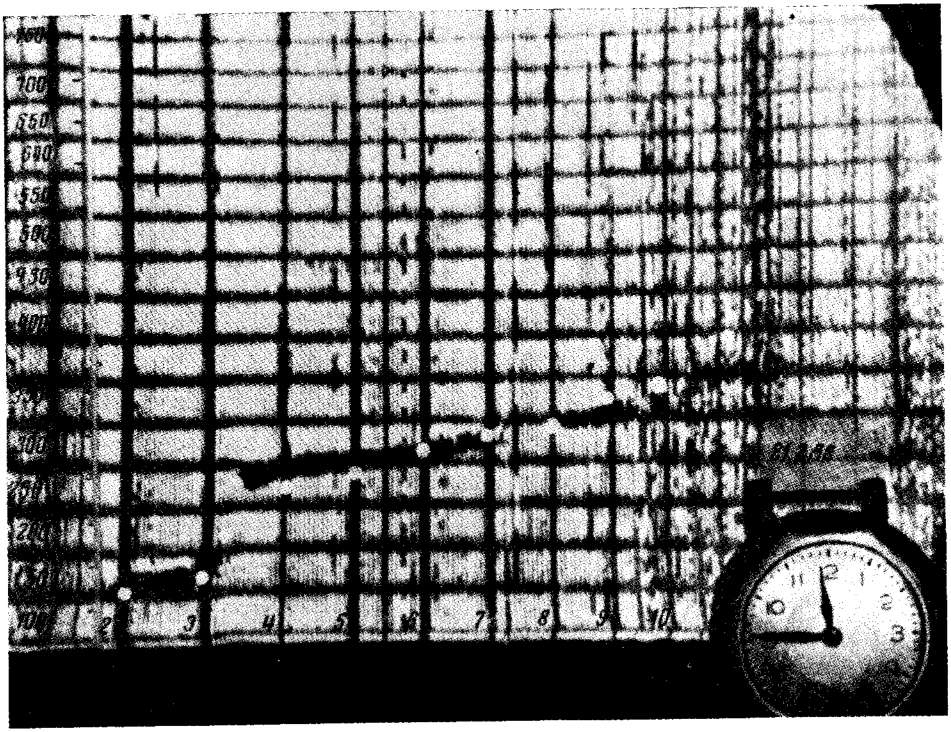
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FIG. 2.

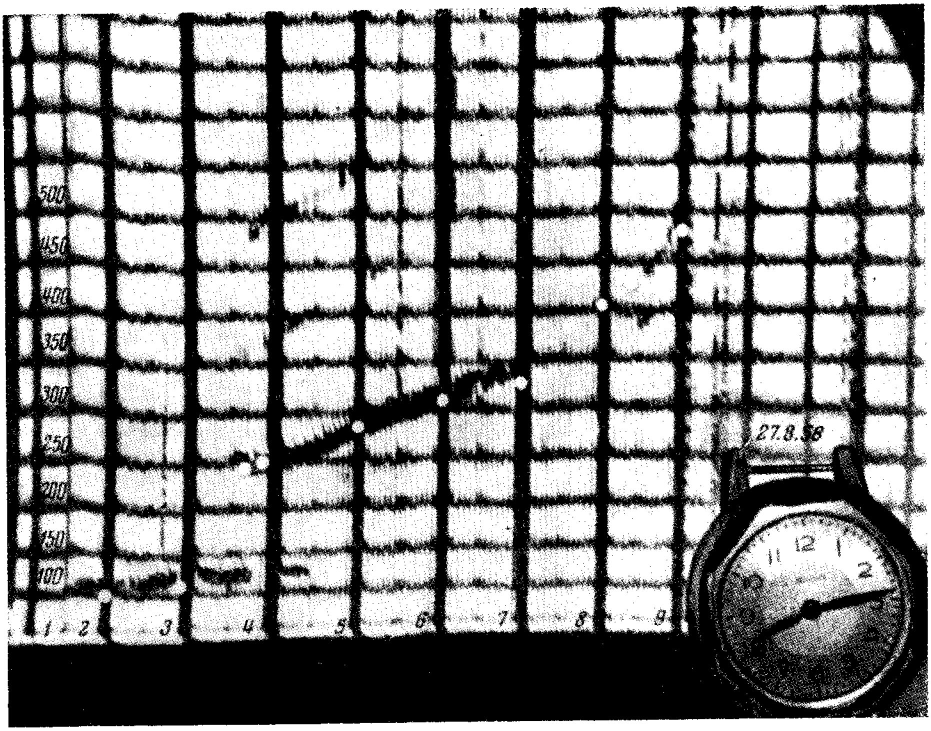
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FIG. 3.

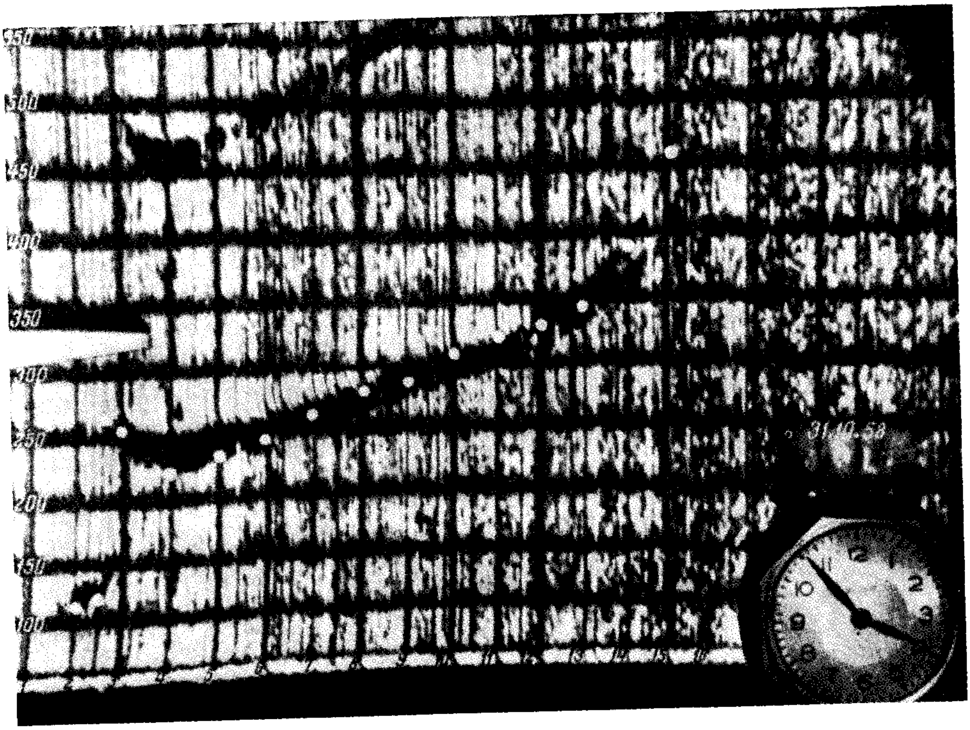

FIG. 4.

measurements into a number of sections in each of which $n_e(h)$ can be taken to be a linear function of h and substituting n as a function of h in form (2) in (1) integral (1) can be taken and (at a given frequency) it is easy to obtain for "operative" thickness of the i-th section:

$$H_{oi} = \frac{2H_i}{x_i - x_{i-1}} (\sqrt{1 - x_{i-1}} - \sqrt{1 - x_i}), \tag{3}$$

where H_i is the true altitude of the i-th section, $x_i = \frac{4\pi e^2}{m\omega^2} n_i(h)$, $x_{i-1} = \frac{4\pi e^2}{m\omega^2} n_{i-1}$ and n_{i-1} and n_i are the concentrations at the beginning and end of the section respectively.

We shall calculate the "operative" height for the frequency ω as follows. The electron concentration distribution curve is broken up into a number of sections to a critical concentration n_k at which radio waves of the frequency ω are reflected. In each of the sections the "operative" thickness is determined by expression (3). The "operative" height for the given frequency is then determined by the expression

$$H_0 = h_0 + \sum_i \frac{2H_i}{x_i - x_{i-1}} (1 - x_{i-1}) - (1 - x_i), \tag{4}$$

where h_0 is the height of the lower boundary of the ionosphere.

In the case when the electron concentration on the i-th section is constant we obtain directly from (1) that

$$H_{oi} = \frac{H_i}{\sqrt{1 - \frac{4\pi e^2}{m\omega^2} n_e}}. \tag{5}$$

The "operative" height values were calculated in this way for a number of frequencies. This collection of values made it possible to draw the corresponding height-frequency curve for each of the $n_e(h)$ distributions shown in Fig. 1.

A number of points from the calculated height-frequency curves were plotted on photographs of the experimental height-frequency curves (Figs. 2, 3 and 4) in order to compare the height-frequency curves calculated in this way with the actual characteristic curves obtained at an ionosphere station in the neighbourhood of the rockets' firing point during the flight of the latter. The photographs clearly show that the calculated curves are close to the actual ones and, in particular, have the same height breaks.

It should be noted that on the experimental height-frequency curve obtained on 27 August 1958 there is a break in the region of a frequency of $f = 3$ Mc/s corresponding to the E_2 layer. This break is not present in the calculated height-frequency curve, the explanation probably being that the $n_e(h)$ curves used in the calculations are smoothed 5 per cent either way(6).

The closeness of the height-frequency curves obtained from rocket measurements of the electron concentration distribution to height-frequency curves obtained by ionospheric sounding from the Earth confirms the reliability of the rocket measurements and the correctness of the conclusions that the region of the ionosphere at altitudes of 100–300 km is a medium with a steady rise in the electron concentration and does not break down into separate layers.

This justifies the assumption of the presence of one (main) n_e maximum which is made when the electron concentration distribution is calculated on the basis of height-frequency curves.

2. COMPARISON OF $n_e(h)$ DISTRIBUTIONS CALCULATED ON BASIS OF HEIGHT-FREQUENCY CURVES WITH THOSE OBTAINED WITH A DISPERSION INTERFEROMETER

The height distribution of electrons was calculated from height-frequency curves by solving the integral equation with Shinn-Kelso coefficients which take account of the Earth's magnetic field. When using this method in the solution of (1) H_o is considered to be a set function of the frequency $H_o = F(f)$ determined by the ionosphere's height-frequency curve.

If the effect of the Earth's magnetic field and collisions of electrons with neutral particles equation (1) is reduced to an Abel integral equation which has the solution (see(7)):

$$H_{fo} = \frac{2}{\pi}\int_0^{f_o} \frac{H_0(f)df}{\sqrt{f_o^2 - f^2}}, \tag{6}$$

where

$$f_o^2 = \frac{e^2 n_e}{\pi m}, \; f^2 = \frac{\omega^2}{4\pi^2}.$$

The use of formula (6) is made difficult in practice because the integral which is part of it is not natural. In order to make it possible to calculate the integral a new variable $\theta = \text{arc}\sin\frac{f}{f_o}$ is introduced. (6) then takes the form

$$H(f_o) = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} H_o(f_o \sin\theta)d\theta. \tag{7}$$

The integration of (7) is carried out numerically by dividing the curve of $H_o(f_o \sin\theta)$ into a series of stages of equal θ width(8). In this case the values of f/f_o which determine the stages selected are called Kelso coefficients.

In order to take account of the effect of the magnetic field Shinn introduced modified coefficients(9) called Shinn-Kelso coefficients. When using Shinn-Kelso coefficients the true altitude corresponding to f_o is given by the equation

$$H(f_o) = \frac{1}{N}\sum_{i=1}^{N} H_o(f_i), \tag{8}$$

where N is the number of stages and f_i is defined by the Shinn-Kelso coefficients.

Since the angle of inclination and magnitude of the magnetic field strength in the area where the U.S.S.R. Academy of Sciences rockets were fired are close to the corresponding values for the Slough Research Station site we used Shinn-Kelso coefficients for five points given in (4)* in the calculations. The calculations were made with an interval of 0·2 Mc/s from the formula

$$h(f_k) = \frac{1}{5}\sum_1^5 h'(f_i),$$

where $h(f_k)$ is the true height of reflexion for the frequency f_k at which the electron concentration n_e (electrons/cm^3) $= 1{\cdot}24 \times 10^{-4} f_k$ (Mc/s), and $h'(f_i)$ is the "operative" altitude for the frequency f_i which is found from f_k by the Shinn-Kelso coefficients.

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* By the time the calculations had been finished paper (10) was published in which coefficients are given (taking account of the magnetic field) for finding concentration distributions from height-frequency curves. The coefficients are given for all stations with a dip of not more than 80°.

The results of the calculations are given in Figs. 5, 6 and 7 in which the electron concentration distributions obtained by rocket measurements⁽⁶⁾ are shown by a dotted line. Figs. 5, 6 and 7 show that the agreement is fairly good.

The divergences between the calculated and experimental curves for the distribution can be explained by the following reasons.

1. By the use of a small number (five) of coefficients.
2. Inaccuracy in reading the "operative" heights and frequencies from the height-frequency curve and equipment errors and signal spread upon reflexion.
3. The absence of frequencies below 1·5 Mc/s in the range of the ionosphere station used (as a result of which lag in the lowest layers is not taken into account).

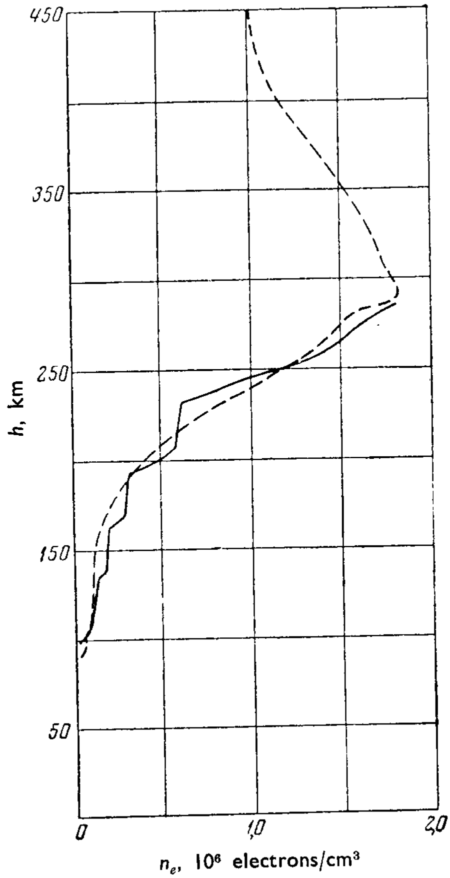

FIG. 5.

4. The presence of some slight deviations from a steady concentration variation between the E and F layers.
5. The averaging (within a range of 5 per cent) of the distribution curve obtained with the dispersion interferometer.

Without making an individual analysis of the errors arising as the result of each of these one can estimate the magnitude of the error on the basis of a comparison of results. The maximum divergence when using five coefficients does not exceed 15 per cent.

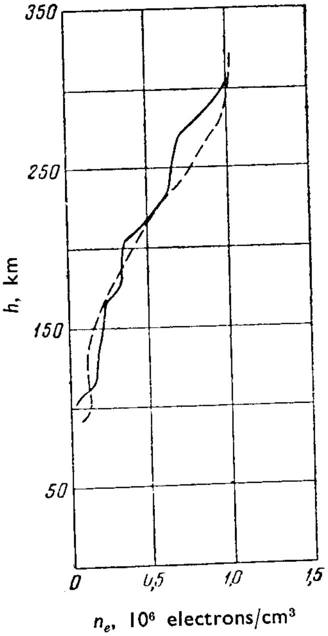

FIG. 6.

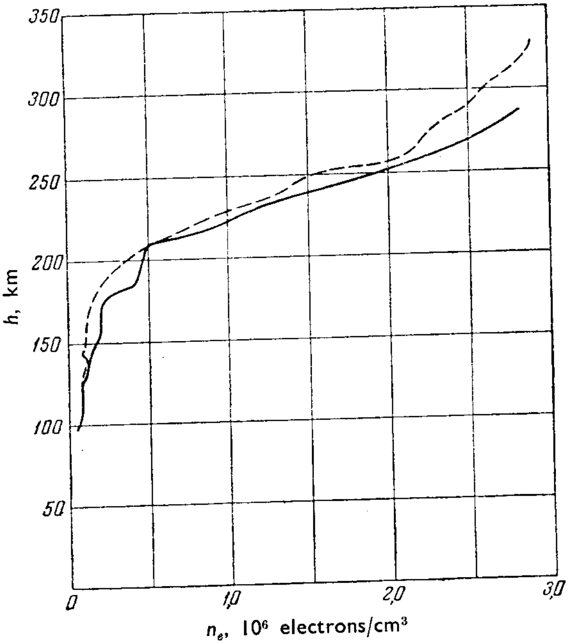

FIG. 7.

It should be pointed out that the sporadic E_s layer was absent in all the cases in which the comparison was made. If this layer is present the errors in calculating the $n_e(h)$ distribution from height-frequency curves may be considerable.

The above discussion shows that it is possible to obtain electron distributions by height from frequency curves up to altitude of the F_2 layer maximum with comparatively small errors by using Shinn-Kelso coefficients.

It is therefore desirable that some of the results (even if only a few) of observations by the ionosphere station network should be published in the form of $n_e(h)$ distributions. This would allow the accumulation of data on $n_e(h)$ variations according to the time of day and year and would enable a number of questions in ionosphere physics to be cleared up.

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REFERENCES

1. F. H. MURRAY and J. B. HOAG. *Phys. Rev.*, **51**, 333 (1937).
2. J. E. JACKSON. *J. Geophys. Res.*, **61**, 101 (1956).
3. G. A. M. KING. *J. Atmos. Terr. Phys.*, **11**, 209 (1957).
4. J. O. THOMAS. *Proc. Met. Radio Engr.* **47**, 162 (1959).
5. B. S. SHAPIRO. *Issledovaniya ionosfery*, No. 5, Akad. Nauk SSSR, p. 93 (1960).
6. K. I. GRINGAUZ and V. A. RUDAKOV. *Isk. Sput. Zemli*, No. 6, 48 (1961).
7. M. P. DOLUKHANOV. *Radio wave propagation* (Rasprostraneniye radiovoln). Moscow, Svyaz'izdat (1952).
8. J. M. KELSO. *J. Geophys. Res.*, **57**, 357 (1952).
9. D. H. SHINN. *J. Geophys. Res.*, **58**, 416 (1953).
10. E. R. SHMERLING and C. A. VENTRICE. *J. Atmos. Terr. Phys.*, **14**, 249 (1959).