A NEW LARGE SCALE INSTABILITY IN A ROTATING STRATIFIED FLOW AND NON LINEAR KINKS

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Abstract. In this presentation, we present linear and non linear structures of a new large scale instability in rotating stratified fluids with small scale forced turbulence. The non linear structures resulting from the saturation of the instability are of helical kinks type. The different solutions that we plotted correspond to solutions linking two different stationary points for different values of the Rayleigh and the Taylor numbers.

1. Introduction

Large scale instabilities are of a great interest in fluid dynamics. They generate vortices which play a fundamental role in turbulence and in transport processes. The generation of this kind of instability has been widely studied, for example in [2], [4], [5]. Many papers as well as the results of numerical modelling are described in detail in review [4]. Several works have shown that a necessary condition for these instabilities to appear is the lack of reflection invariance or parity invariance [2]. In fluid dynamics, the widespread mechanism used to break this symmetry is the helicity $\vec{v} \cdot rot\vec{v} \neq 0$. However, it has been shown that the helicity itself is not enough to generate these large scale structure, and others factors are necessary like for example temperature gradient [5]. It has also been shown in a previous work [5] that the method of multi-scale development can lead to the occurrence of large scale instabilities in helical stratified turbulence.

In this work, we find a new large scale instability in rotating stratified fluids with small scale forced turbulence. The helicity can be used in an explicit way [5] as well as in a internal way. We use the Coriolis force and the buoyancy to create naturally the internal helicity which is required to break the symmetry of the flow. That is what allows us to use a force which does not have any particular properties (especially it is nonhelical and it does not lack parity invariance). The force only maintains turbulent fluctuations. Also, we solve the non linear instability equations. As a result of saturation, the non linear structures appearing are rotational kinks.

2. The main equations and formulation of the problem

Let's consider the equations for the motion of an incompressible fluid with a constant temperature gradient in the Boussinesq approximation using dimensionless variables:

$$\frac{\partial \vec{V}}{\partial t} + R\left(\vec{V}\cdot\nabla\right)\vec{V} - \Delta\vec{V} + D\vec{l}\times\vec{V} = -\nabla P + RaT\vec{l} + \vec{F}_0 \qquad (1)$$

$$\frac{\partial T}{\partial t} - \Delta T = -V_z - R\left(\vec{V}\cdot\nabla\right)T \qquad (2)$$

$$\nabla\cdot\vec{V} = 0,$$

where $\vec{l} = (0,0,1)$, β is the thermal expansion coefficient, $A = dT_0/dz$ is the constant equilibrium gradient of the temperature, $\rho_0 = Const$, and $\nabla T_0 = A\vec{l}$. R and D are respectively the Reynolds number and the scare root of the Taylor numbers. $\Pr$ represents the Prandtl number and Ra is the Rayleigh number. We also introduce a dimensionless temperature T.

We will consider as a small parameter of an asymptotic development the Reynolds number R. Let us denote the small scale variables by $x_0 = \left(\vec{x}_0, t_0\right)$, and the large scale ones by $X = \left(\vec{X}, T\right)$.

The small scale partial derivative operation $\frac{\partial}{\partial x_0^i}, \frac{\partial}{\partial t_0}$, and the large scale ones $\frac{\partial}{\partial \vec{X}}, \frac{\partial}{\partial T}$ are written, respectively, as $\partial_i, \partial_t, \nabla_i$ and ∂_T. To construct a multi-scale asymptotic development we follow the method which is proposed in [2].

3. The multi-scale asymptotic development

Following [1], let us look for the solutions to Equations (1) and (2) in the following form:

$$\vec{V}\left(\vec{x},t\right) = \frac{1}{R}\vec{W}_{-1}\left(X\right) + \vec{v}_0\left(x_0\right) + R\vec{v}_1 + R^2\vec{v}_2 + R^3\vec{v}_3 + \cdots \qquad (3)$$

$$T\left(\vec{x},t\right) = \frac{1}{R}T_{-1}\left(X\right) + T_0\left(x_0\right) + RT_1 + R^2T_2 + R^3T_3 + \cdots \qquad (4)$$

The scale relation is the following: $\vec{X} = R^2\vec{x}_0$ and $T = R^4 t_0$. From this we get the main secular equation at order $\mathrm{O}\left(R^3\right)$:

$$\partial_T W_{-1}^i - \Delta W_{-1}^i + \nabla_k\left(\overline{v_0^k v_0^i}\right) = -\nabla_i \overline{P}_1 \qquad (5)$$

$$\partial_T T_{-1} - \Delta T_{-1} = \nabla_k\left(\overline{v_0^k T_0}\right) = 0 \qquad (6)$$

4. Calculations of the Reynolds stresses

The essential equation for finding the nonlinear alpha-effect is equation (\ref{68.4}). In order to obtain these equations in closed form, we need to calculate the Reynolds stresses $\nabla_k\left(\overline{v_0^k v_0^i}\right)$.

The external force can be chosen in a general $3D$ form like for example:

$$\vec{F}_0 = f_0\left(\vec{i}\cos\varphi_1 + \vec{j}\cos\varphi_2 + \vec{k}\cos\varphi_2\right), \qquad (7)$$

where

$$\varphi_1 = k_0 z - \omega_0 t, \quad \varphi_2 = k_0 x - \omega_0 t, \qquad (8)$$

However, it can be shown that only one component of this force is responsible for displaying large scale instability, which is the (x,z) plan component. The force can then be reduced to:

$$\vec{F}_0 = f_0\left(\vec{i}\cos\varphi_1 + \vec{k}\cos\varphi_2\right) \tag{9}$$

Below are the expression of the Reynolds stresses as calculated in [1]:

$$T_{(1)}^{31} = \frac{D^2\left[2 + Ra - 2\left(1-W_1\right)^2\right]}{\Xi_{(1)}}$$

$$T_{(2)}^{31} = \frac{Ra\left[2 + D^2 - 2\left(1-W_2\right)^2\right]}{\Xi_{(2)}}$$

$$T_{(1)}^{32} = \frac{-D^3\left[Ra + 2\left[1-\left(1-W_1\right)^2\right]\right]}{\Xi_{(1)}}$$

$$T_{(2)}^{32} = \frac{-DRa\left[2 + D^2 - 6\left(1-W_2\right)^2\right]}{\Xi_{(2)}}$$

Where

$$\Xi_{(1),(2)} = 2\left[D^4 + Ra^2 + 2D^2Ra + 4\left[1+\left(1-W_{1,2}\right)^2\right]^2 + \right.$$
$$\left. +\left(2D^2 + 2Ra\right)\left[2-2\left(1-W_{1,2}\right)^2\right]\right]\left[1+\left(1-W_{1,2}\right)^2\right]$$

5. Large scale instability

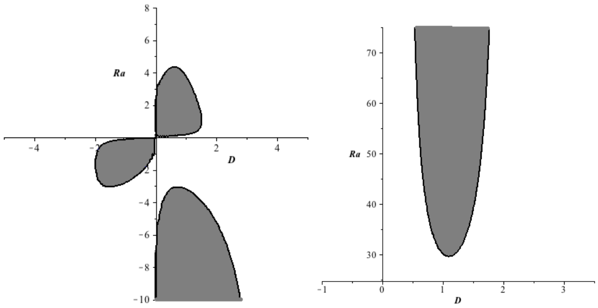
Let us write down in the explicit form the equations for nonlinear instability:

$$\partial_T W_1 + \nabla_Z T_{(1)}^{31} + \nabla_Z T_{(2)}^{31} = \Delta W_1 \tag{10}$$

$$\partial_T W_2 + \nabla_Z T_{(1)}^{32} + \nabla_Z T_{(2)}^{32} = \Delta W_2, \tag{11}$$

where the components $T_{(1)}^{31}$, $T_{(2)}^{31}$, $T_{(1)}^{32}$ and $T_{(2)}^{32}$ of the Reynolds stress tensor are as defined in the previous section.

We show in a previous paper [1] that the linear instability can appear for specific values of D and Ra. We show below two figures representing the area (in grey) of the plane (D, Ra) for which the discriminant is negative, this means that an instability can appear. The first plot shows the conditions for a negative temperature gradient and the second plot, for a positive one.

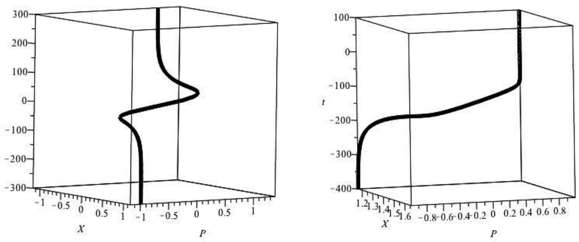


(a) (b)

Figure 1: Instability condition with (a) negative temperature gradient (b) positive temperature gradient

6. Instability saturation and non linear structures

The increase of W_1 and W_2 leads to the saturation of the instability. As a result of the development and stabilization of the instability, non linear structures appear. The study of these structures is of interest. Integrating this system using simple numerical tools in the stationary case allow us to display non linear structures.



(c) (d)

Figure 2: Helical Kinks with $D=1.5$, $Ra=1.9$, $C_1=0.05$, $C_2=0.01$, (c) between two nodes (d)between a node and a hyperbolic point.

7. Conclusion and discussion of the results

In this presentation, we first exposed the main results of the linear stage of a large scale instability in rotating and stratified flow. Then we show nonlinear large scale structures displayed by such a flow. These solutions show that in the non linear stage, the instability saturation leads to specific velocity profiles (helical kinks) for which the velocity tends to be constant for large values of Z. These structures are of helical type and are the result of the saturation of the instability. Since the phase portrait doesn't contain any elliptic stationary point there is no periodical solution but only rotational kink solutions.

References

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